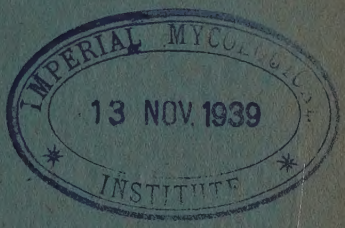
COMPTES RENDUS

(DOKLADY)

DE L'ACADÉMIE DES SCIENCES DE L'URSS

NOUVELLE SÉRIE

1939



VOLUME XXIV

№ 2

ÉDITION DE L'ACADÉMIE DES SCIENCES DE L'URSS
MOSCOU

AVIS AUX AUTEURS

Les «Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS» paraissent tous les dix jours, neuf numéros formant un volume d'environ 800 pages pourvu d'une table des matières.

Les «Comptes Rendus de l'Académie des Sciences de l'URSS» publient des communications courtes précisant l'essentiel des travaux plus volumineux en préparation. Ces articles, ne dépassant pas comme règle dix mille signes typographiques, sont donnés en l'une des trois langues: français, anglais, allemand. Ils embrassent les sciences mathématiques, physiques, naturelles et techniques, reproduisant les résultats de recherches scientifiques actuellement en cours ou à peine achevées qui représentent les plus récentes données de l'investigation scientifique.

Les auteurs ont droit chacun à 25 extraits gratuits.

Les manuscrits doivent être adressés à la Rédaction des «Comptes Rendus», Moscou 19, Volchonka, 14, chambre 211.

20 Juillet

COMPTES RENDUS

(DOKLADY)

DE L'ACADÉMIE DES SCIENCES DE L'URSS

COMITÉ DE RÉDACTION

A. Borissiak, de l'Académie, **A. Fersman**, de l'Académie, **A. Frumkin**, de l'Académie, **A. Richter**, de l'Académie, **S. Wawilow**, de l'Académie

NOUVELLE SÉRIE

7-me année

Paraissant tous les dix jours

1939

Volume XXIV, № 2

TABLE DES MATIÈRES

Pages

MATHÉMATIQUES

Alexander Kurosch (А. Г. Курош). Lokal freie Gruppen 99

M. Keldych (М. Келдыш) et M. Lavrentieff (М. Лаврентьев). Sur une évaluation pour la fonction de Green 102

P. E. Dubuque (П. Е. Дюбюк). Sur les sous-groupes invariants dans un groupe fini 104

P. V. Soloviev (П. В. Соловьев). Fonctions de Green des équations paraboliques 107

THÉORIE DE L'ÉLASTICITÉ

P. Riz (П. М. Риз). On Deformations of Bars with Slightly Curved Axis 110

MÉCANIQUE CÉLESTE

G. Badalian (Г. Бадалян). Sur la simplification de l'équation de la trajectoire dans le problème de deux centres fixes 114

PHYSIQUE

V. G. Khlopin (В. Г. Хлопин), Member of the Academy, **M. A. Pasvick-Khlopina (М. А. Пасвик-Хлопина)** and **N. F. Volkov (Н. Ф. Волков).** Division of Uranium Nuclei under the Action of Neutrons and the Problem of the Existence of Trans-Uraniums 118

A. T. Ashtcheulov (А. Т. Ащеулов). On the Nature of the Anomalous Physical Properties of Thin Silver Films 122

E. F. Gross (Е. Ф. Гросс) and **A. I. Raskin (А. И. Раскин).** Nature of the Low-Frequency Raman Spectrum and the Spectrum of the Hexamethylbenzene Crystal 126

L. I. Roussinow (Л. И. Русинов) and **A. A. Yusephovich (А. А. Юзефович).** Energy of Conversion Electrons Arising in the Transformation of Bromine Isomers 129

V. L. Ginsburg (В. Л. Гинзбург). Some Contribution to Quantum Electrodynamics. III 131

S. E. Haykin (С. Е. Хайкин), L. P. Lissovsky (Л. П. Лисовский) and A. E. Solomonovich (А. Е. Соломонович). On «Dry Friction» Forces 135

PHYSIQUE TECHNIQUE

K. M. Kossikov (К. М. Косиков). Determination of Minimum Working Wave Lengths from Measurements of the Ionosphere 140

PHYSIQUE DU GLOBE

V. G. Dybchenko (В. Г. Дыбченко). A More Precise Determination of Humidity from the Records of Aerological Sounds 143

CHIMIE

A. V. Babajeva (А. В. Бабаева). On the Oxidation of Complex Compounds of Platinum with Hydrogen Peroxide 145

A. I. Pankratov (А. И. Панкратов). Change in the Drop of Potential at the Air/Solution Interface as Related to the Age of the Latter . . . 149

N. D. Tomashov (Н. Д. Томашов). Change in the Rate of Solution of Copper during Anodic and Cathodic Polarizations. Difference-Effect, Protective-Effect 152

GÉOLOGIE

J. K. Dzevanovsky (Ю. К. Дзевановский). Section of Cambrian Deposits of the Northern Margin of the Aldan Platform and of the Adjoining Folded Region 156

V. N. Makhaev (В. Н. Махаев). A Contribution to the Stratigraphy of the Cambrian of the Maya River Basin 159

MINÉRALOGIE

G. V. Gvakharia (Г. В. Гвахария). Datolite from the Soganlug Talus in the Neighbourhood of the Town of Tbilissi 162

M. N. Shkabara (М. Н. Шкабара) and E. A. Sturm (Е. А. Штурм). Datolite from the Volcanic Group of the Karadag in the Crimea . . 166

GÉNÉTIQUE

P. F. Rokizky (П. Ф. Рокицкий). Influence of Homozygosity and Heterozygosity upon Quantitative Characters 171

V. A. Koroleva (В. А. Королева). Interspecific Hybridization in the Genus *Taraxacum* 174

T. K. Enin (Т. К. Енин). Results of an Analysis of Segregation of Tomato Hybrids Conducted for Each Family Separately 177

M. A. Rosanova (М. А. Розанова). Evolution of Cultivated Raspberry 179

V. I. Patrushev (В. И. Патрушев). Physiological Differences in Draft-Horses as Correlated with Their Performance 182

MICROBIOLOGIE

V. Zazhurilo (В. Зажурило) and P. Schwartz (П. Шварц). Anatomical and Cytological Modifications in Cereals Attacked by Mosaic Virus . . 185

AGROBIOLOGIE

P. Schwartz (П. Шварц). A Simple Method for Determining the Infection of Wheat Seeds with *Ustilago tritici* 189

PHYSIOLOGIE VÉGÉTALE

D. J. Vakulin (Д. Я. Вакулин) and M. J. Reutman (М. Я. Ройтман). Oil Constants of Various Forms of *Lallemantia* 192

AGROPHYSIOLOGIE

I. M. Vassiliev (И. М. Васильев). Growth of Wheat of Varying Frost Resistance during the Hardening Period 195

CYTOLOGIE

V. Glotov (В. Глотов). Dimorphism of the Caryotype in *Chrysanthemum coronarium* 199

MORPHOLOGIE EXPÉRIMENTALE

G. V. Lopashov (Г. В. Лопашов). On the Character of Cell Changes in Neural Plate Induction 205

Rédacteur en chef A. Fersman. Rédacteur S. Bernstein.

En composition 23/VI 1939. En impression 23/VI 1939. Nombre de feuilles d'impression 7. Nombre de caractères par feuille d'impression 65.000. Pages 97—208. Format du papier 72×110 cm Tirage 1 000. Glavlit A 15927. Numéro d'édition 2080. Numéro typographique 1395.

Imprimerie № 16, Moscou, Trekhproudnij péréoulok, 9.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIV, № 2

MATHEMATIK

LOKAL FREIE GRUPPEN

Von ALEXANDER KUROSCH

(Vorgelegt von O. J. Schmidt, Mitglied der Akademie, d. 19. V. 1939)

Eine Gruppe G soll lokal frei heissen, wenn jede endliche Teilmenge von G eine freie Untergruppe erzeugt. Zu dieser Klasse von Gruppen gehören u. a.: die freien Gruppen von beliebiger Mächtigkeit (1), die additive Gruppe aller rationalen Zahlen und ihre Untergruppen und auch die in der Arbeit (2) konstruierte Gruppe T.

Jede Untergruppe einer lokal freien Gruppe ist selbst lokal frei. Alle von 1 verschiedenen Elemente einer lokal freien Gruppe sind von unendlicher Ordnung. Die Vereinigung einer zunehmenden Folge von lokal freien Gruppen ist selbst lokal frei. Im Falle der abzählbaren Gruppen kann man diese Behauptung folgendermassen umkehren.

Jede abzählbare lokal freie Gruppe ist die Vereinigung einer zunehmenden Folge von freien Gruppen mit endlichvielen Erzeugenden.

Die Frage über die Existenz der unabzählbaren lokal freien Gruppen, die keine Vereinigung einer zunehmenden Folge von freien Gruppen sind, bleibt offen.

Das freie Produkt einer beliebigen Menge von lokal freien Gruppen ist selbst lokal frei.

Beweis. Ist $G=\prod_{\alpha}^{*} H_\alpha$ und ist M eine endliche Teilmenge von G, so nehmen wir alle Elemente eines jeden der Faktoren H_α zusammen, die in den Worten, welche die Elemente von M darstellen, enthalten sind. Diese endliche Teilmenge von H_α erzeugt in H_α eine freie Untergruppe H'_α. Das Produkt $\prod_{\alpha} H'_\alpha$ ist eine freie Gruppe und daher ist seine Untergruppe $\{M\}$ auch frei.

Eine lokal freie Gruppe G besitzt einen endlichen Rang, und zwar den Rang n, wo n die kleinste natürliche Zahl ist mit der Eigenschaft, dass jede endliche Teilmenge von G in einer freien Untergruppe mit n freien Erzeugenden enthalten ist. Jede abzählbare lokal freie, aber nicht freie Gruppe vom Range n ist die Vereinigung einer zunehmenden Folge von freien Gruppen mit n freien Erzeugenden. Man kann diese Gruppe im Falle $n>1$ auch wie die Vereinigung einer zunehmenden Folge von freien Gruppen mit m Erzeugenden für jedes $m>n$ darstellen, und sogar wie die Vereinigung einer zunehmenden Folge von freien Gruppen mit unbegrenzt wachsender Anzahl der freien Erzeugenden.

Die Vereinigung einer zunehmenden Folge von unabzählbaren freien Gruppen ist keine Gruppe von endlichem Rang.

Beweis. Es sei G eine lokal freie Gruppe vom Range n, die mit der Vereinigung einer zunehmenden Folge von freien Gruppen $S_1 \subset S_2 \subset \subset \ldots \subset S_k \subset \ldots$ übereinstimmt, und M — eine endliche Teilmenge von G, die zu keiner Untergruppe mit einer Anzahl der Erzeugenden kleiner als n gehört. Es gibt dann eine Zahl k und eine unabzählbare Teilmenge P von G derart, dass jede Menge, die aus M und aus einem beliebigen Element von P besteht, schon in der Gruppe S_k in einer freien Untergruppe mit n Erzeugenden enthalten ist. Man kann alle Elemente von M durch endlichviele Erzeugende von S_k darstellen, d. h. M gehört zu einem abzählbaren freien Faktor A von S_k. Da die Menge P unabzählbar ist, so kann man in P ein zu A nicht gehörendes Element b finden. Die Menge M', die aus M und b besteht, gehört zu einer Untergruppe B mit n Erzeugenden, $B \subset S_k$. Nach dem Untergruppensatz für freie Produkte ([3]) ist der Durchschnitt $B \cap A$ ein freier Faktor von B. Dieser Durchschnitt enthält aber die Menge M und besitzt daher nicht weniger als n freie Erzeugende, was zur Ungleichheit $B \neq B \cap A$ im Widerspruch steht.

Ist eine lokal freie Gruppe G von endlichem Rang n auf eine lokal freie Gruppe $\overline{G}$ homomorph (aber nicht isomorph) abgebildet, so ist der Rang von $\overline{G}$ endlich und kleiner als n.

Beweis. Wir nehmen in $\overline{G}$ eine beliebige endliche Teilmenge $\overline{M}$, und in G eine solche endliche Teilmenge M, die ein Urbild zu jedem Element von $\overline{M}$ und überdies noch ein von 1 verschiedenes Element, dessen Bild in $\overline{G}$ gleich 1 ist, enthält. Die Menge M gehört zu einer Untergruppe A mit n freien Erzeugenden. Das Bild dieser Untergruppe ist eine freie Untergruppe $\overline{A}$ von $\overline{G}$ und, da die Abbildung A auf $\overline{A}$ homomorph ist, ist die Anzahl der freien Erzeugenden von $\overline{A}$, nach dem Satz von Whitehead ([4]), kleiner als n.

Daraus folgt die Verallgemeinerung eines Satzes von Magnus ([5]) über freie Gruppen:

Jede lokal freie Gruppe von endlichem Rang ist zu keiner ihrer echten Faktorgruppen isomorph.

Der Rang des freien Produktes von zwei lokal freien Gruppen von endlichem Rang stimmt mit der Summe der Rangen dieser Faktoren überein.

Beweis. Es sei $G = A \star B$, A vom Range r, B vom Range s. Ist M eine endliche Teilmenge von G, so kann man ihre Elemente durch eine endliche Anzahl von Elementen von A und B darstellen, d. h. man kann derartige freie Untergruppen $A' \subset A$, $B' \subset B$ mit r, bzw. s freien Erzeugenden finden, dass ihr freies Produkt schon die Menge M enthält. Der Rang der Gruppe G ist daher nicht grösser als $r+s$. Wir nehmen andererseits aus der Untergruppe A eine endliche Teilmenge M_1, die zu keiner freien Untergruppe mit einer Anzahl von freien Erzeugenden kleiner als r gehört; aus B nehmen wir eine entsprechende Teilmenge M_2. Jede die Mengen M_1 und M_2 enthaltende Untergruppe C von G ist nach ([3]) in ein freies Produkt so zerlegbar, dass u. a. beide Durchschnitte $C \cap A$, $C \cap B$ freie Faktoren sind. Die Anzahl der freien Erzeugenden von C ist daher nicht kleiner als $r+s$.

Um die Frage über die Existenz der frei unzerlegbaren lokal freien Gruppen, die von den Gruppen vom Range 1 verschieden sind, zu entscheiden, beweisen wir zuerst ein Lemma.

Das freie Produkt von zwei lokal freien Gruppen stimmt mit keinem von seinen Normalteilern, die von einem Element erzeugt sind, überein.

Beweis. Sei $G = A \star B$ und möge der von einem Element c erzeugte Normalteiler mit G übereinstimmen. Es ist offenbar $c \not\subset A$. Wir wählen $a \subset A$, $a \neq 1$. Dieses Element ist das Produkt endlichvieler mit c konjugierter Elemente. Die Elemente von A und B, durch die alle diese mit c konjugierten Elemente und das Element c selbst dargestellt sind, erzeugen in A und B freie Untergruppen A', bzw. B'; dabei gehört das Element a schon im Produkte $A' \star B'$ zum von c erzeugten Normalteiler. Da $a \not\subset A'$, so müsste, nach dem Freiheitssatz (6), die Hinzufügung der Relation $c = 1$ die Untergruppe A' frei lassen, in der Tat aber erhalten wir $a = 1$.

Ist eine freie Gruppe S mit freien Erzeugenden a und b_α, wo α eine Indexmenge $\mathfrak{M}$ durchläuft, gegeben, so erweisen sich die Elemente a und $c_\alpha = b_\alpha\, a b_\alpha$, $\alpha \subset \mathfrak{M}$ als freie Erzeugende einer echten Untergruppe H. Wir nehmen jetzt eine zunehmende Folge von freien Gruppen, die ineinander genau so eingebettet sind, wie H in S. Ist

$$S_1 \subset S_2 \subset \ldots \subset S_k \subset \ldots$$

diese Folge und besitzt S_k, $k = 1, 2, \ldots$, die freien Erzeugenden a und $b_{k\alpha}$, $\alpha \subset \mathfrak{M}$, so setzen wir

$$b_{k\alpha} = b_{k+1,\alpha}^{-1}\, a\, b_{k+1,\alpha}.$$

Die Vereinigung dieser Folge stimmt offenbar mit ihrem vom Element a erzeugten Normalteiler überein, und ist daher frei unzerlegbar. *Es gibt also frei unzerlegbare lokal freie Gruppen einer beliebigen unendlichen Mächtigkeit.*

Im Falle der Gruppen von endlichem Rang bemerken wir noch ergänzend, dass wenn wir mit D_n die Gruppe bezeichnen, die in der obendargelegten Weise für eine endliche, aus n Elementen bestehende, Menge $\mathfrak{M}$ konstruiert ist, so hat offenbar die Gruppe D_1 den Rang 2, und der Rang der Gruppe D_n ist nicht grösser als $n+1$. Dieser Rang ist tatsächlich gleich der Zahl $n+1$: man kann die Gruppe D_n auf die Gruppe D_{n+1} homomorph abbilden. *Es gibt also frei unzerlegbare abzählbare lokal freie Gruppen von einem beliebigen endlichen Range.*

Eingegangen
d. 21. V. 1939.

ZITIERTE LITERATUR

[1] O. Schreier, Hamb. Abhandl., 5, 161—183 (1927). [2] А. Курош, Матем. сб., 2, 995—1001 (1937). [3] A. Kurosch, Math. Ann., **109**, 647—660 (1934). [4] J.K.C. Whitehead, Ann. of Math., **37**, 782—800, § 4 (1936). [5] W. Magnus, Math. Ann., **111**, 259—280 (1935). [6] W. Magnus, J. f. reine u. angew. Math., **163**, 141—165 (1930).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIV, № 2

MATHÉMATIQUES

SUR UNE ÉVALUATION POUR LA FONCTION DE GREEN

Par M. KELDYCH et M. LAVRENTIEFF

(Présenté par S. L. Soboleff, de l'Académie, le 5. V. 1939)

1. Soit D un domaine borné par une surface fermée et unie S dans l'espace de trois dimensions (x, y, z). Désignons par $G(P, Q)$ la fonction de Green pour le domaine D ayant pour pôle le point Q. Dans plusieurs questions de la théorie du potentiel l'évaluation suivante de la fonction de Green joue un rôle essentiel:

$$G(P, Q)] < \frac{A\sigma s}{PQ^3} \tag{1}$$

où A est une constante qui ne dépend que de la forme du domaine D et σ et s sont respectivement les distances des points P et Q de la surface S.

Pour le cas où D est une sphère, l'évaluation (1) est obtenue immédiatement de l'expression de G.

Rosenblatt a démontré que (1) a encore lieu quand S possède une courbure bornée (1). Nous nous proposons de démontrer que (1) subsiste quand S est une surface arbitraire de Liapounoff *).

2. La démonstration est basée sur une construction auxiliaire. Fixons deux nombres positifs $\rho < \frac{1}{2}$ et $\alpha < 1$ et désignons par $P_k(t)$ la solution de l'équation de Legendre d'ordre k régulière et égale à 1 pour $t = 1$. Soit ensuite $r = \sqrt{x^2+y^2+z^2}$ et θ — l'angle formé par le rayon-vecteur du point (x, y, z) avec l'axe z. Désignons par $T(\alpha, \rho)$ le corps contenant les points de la partie positive de l'axe z et borné par les surfaces

$$r\cos\theta - r^{1+\alpha} P_{1+\alpha}(\cos\theta) = 0 \tag{2}$$

$$r = \rho. \tag{3}$$

La partie de la surface (2) appartenant à la frontière de T sera nommée surface latérale de T. Remarquons qu'au voisinage de l'origine la surface (2) est de la forme

$$z = -|P_{1+\alpha}'(0)|(1+\varepsilon)(x^2+y^2)^{\frac{1+\alpha}{2}}$$

où ε tend vers zéro quand le point (x, y) tend vers l'origine.

*) Une surface S est appelée surface de Liapounoff, si quels que soient deux points M_1 et M_2 de la surface, l'angle ψ entre les normales à S de M_1 et M_2 vérifie l'inégalité $\psi < K(\overline{M_1M_2})^\nu$, où K et ν, $0 < \nu < 1$ sont des constantes données.

Construisons à l'intérieur de T une fonction harmonique

$$W_{\alpha,\rho}(r,\theta)=\frac{A}{\rho}(r\cos\theta-r^{1+\alpha}P_{1+\alpha}(\cos\theta)-\frac{B}{\rho^2}r^2P_2(\cos\theta). \tag{4}$$

Il est facile à voir que l'on peut choisir les nombres $\rho_0<\frac{1}{2}$, A et B de telle manière que pour $\rho<\rho_0$ la fonction $W_{\alpha,\rho}$ vérifie les inégalités: $W_{\alpha,\rho}\geqslant 0$ sur la surface latérale de T et $W_{\alpha,\rho}\geqslant 1$ sur la partie restante de la frontière de T.

Remarquons d'ailleurs que dans le corps T sur l'axe z on a

$$W<(A+B)\frac{r}{\rho}. \tag{5}$$

3. Soit D un domaine borné par une surface de Liapounoff S d'exposant ν. Désignons par $T(\alpha,\rho,M)$ un corps congruent à $T(\alpha,\rho)$ ayant son sommet au point M de la surface S et la normale intérieure au point M coïncidant avec la normale intérieure à S. En posant $\alpha<\nu$ nous pouvons choisir un nombre ρ_1 indépendant de M et tel que la surface latérale du corps $T(\alpha_1,\rho_1,M)$ soit située hors de D.

Considérons la fonction de Green $G(P,Q)$. Il est évident que

$$G(P,Q)<\frac{1}{\overline{PQ}}. \tag{6}$$

Démontrons que

$$G(P,Q)<A_1\frac{\sigma}{\overline{PQ}^2} \tag{7}$$

où A_1 est une constante qui ne dépend que de la forme du domaine D.

Soit d le diamètre de D; il suffit de démontrer (7) pour le cas où $\sigma<\frac{\rho}{4d}\overline{PQ}$. Soit M le point de S qui est le plus proche de P_0 et $\rho=\frac{\rho_1}{2d}\overline{P_0Q}$. Considérons le corps $T(\alpha,\rho,M)$ et la fonction correspondante $W_{\alpha,\rho,M}$. En tenant compte de ce que $G(P,Q)=0$ sur la surface de S et qu'aux points de la sphère $MP=\rho$, en vertu de (6) et du choix de ρ on a

$$G(P,Q)<\frac{4d}{\rho_1}\frac{1}{\overline{P_0Q}};$$

nous concluons en vertu du principe du maximum qu'aux points du domaine D appartenant au corps $T(\alpha,\rho,M)$ on a

$$G(P,Q)<\frac{4d}{\rho_1}\frac{W_{\alpha,\rho,M}}{\overline{P_0Q}}$$

et en vertu de (5) au point P_0 situé sur l'axe du corps $T(\alpha,\rho,M)$

$$G(P,Q)<\frac{4d(A+B)}{\rho_1}\frac{\sigma}{\rho\overline{P_0Q}}=\frac{16d^2(A+B)}{\rho_1^2}\frac{\sigma}{\overline{P_0Q}^2}$$

ce qui prouve l'inégalité (7). En partant de (7) et en considérant $G(P,Q)$ comme fonction Q, on obtient au moyen des mêmes raisonnements l'inégalité

$$G(P,Q)<\frac{1024d^5(A+B)^2}{\rho_1^5}\frac{s\sigma}{\overline{PQ}^3}. \tag{8}$$

Institut Mathématique de Stekloff.
Académie des Sciences de l'URSS.

Manuscrit reçu
le 27. V. 1939.

LITTÉRATURE CITÉE

[1] A. Rosenblatt, C. R., 201.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIV, № 2

MATHÉMATIQUES

SUR LES SOUS-GROUPES INVARIANTS DANS UN GROUPE FINI

Par P. E. DUBUQUE

(Présenté par O. J. Schmidt, de l'Académie, le 19. V. 1939)

Le but de la présente Note est de préciser et de généraliser les résultats obtenus par l'auteur dans ses articles «Sur le normalisateur d'un élément dans un groupe fini» (1) et «Représentations monomiales et critères pour qu'un groupe soit non simple» (2).

En appliquant les notations que nous avons utilisées dans les articles cités nous pouvons énoncer les théorèmes suivants sur le normalisateur d'un élément dans un groupe fini.

Théorème 1. *Soit A un élément d'ordre p^k d'un certain groupe $\mathfrak{G}$ (p étant un nombre premier). Soit $\mathfrak{P}$ un sous-groupe d'indice φ du quasi-normalisateur $\mathfrak{N}_{A^{p^i}}^{(k-i-1)}$ de l'élément A^{p^i}. Si φ n'est pas divisible par p^{i+1} et si chaque élément $\mathfrak{P}$ conjugué à A^z est de la forme A^{mz}, où $m \equiv 1 \pmod p$, alors le rapport des ordres des normalisateurs des éléments $A^{p^{i+1}}$ et A^{p^i} n'est pas divisible par p^2.*

Théorème 2. *Soit $\mathfrak{P}$ le p-sous-groupe de Syloff du $(k-1)$-ième quasi-normalisateur d'un élément A d'ordre p^k du groupe $\mathfrak{G}$ (p étant un nombre premier). Si chaque élément $\mathfrak{P}$ conjugué à A^z est de la forme A^{mz}, où $m \equiv 1 \pmod p$, alors le rapport des ordres des normalisateurs des éléments $A^{p^{k-1}}$ et A n'est pas divisible par p^k.*

Théorème 3. *Soit A un élément d'ordre p^k d'un groupe $\mathfrak{G}$ (p étant un nombre premier). Si chaque élément du quasi-normalisateur $\mathfrak{N}_{A^{p^i}}^{(k-i-1)}$ de l'élément A^{p^i} conjugué à A^{p^i} est de la forme A^{mp^i}, où $m \equiv 1 \pmod p$, alors le rapport des ordres des normalisateurs des éléments $A^{p^{i+1}}$ et A^{p^i} n'est pas divisible par p^2.*

Théorème 4. *Soit A un élément d'ordre p^k d'un groupe $\mathfrak{G}$ (p étant un nombre premier impair). Soit $\mathfrak{P}$ le p-sous-groupe de Syloff du $(k-1)$-ième quasi-normalisateur de l'élément A. Si chaque élément du groupe $\mathfrak{P}$ vérifiant la condition $P^p \subset \{A\}$ et conjugué à A^z est de la forme A^{mz}, où $m \equiv 1 \pmod p$, alors le rapport des ordres des normalisateurs des éléments $A^{p^{k-1}}$ et A n'est pas divisible par $p^{k-\lambda_0+1}$.*

Théorème 5. *Soit A un élément d'ordre p^k d'un groupe $\mathfrak{G}$ (p étant un nombre premier). Sois $\mathfrak{P}$ le p-sous-groupe du normalisateur du groupe cyclique $\{A\}$. Si chaque élément du groupe $\mathfrak{P}$ vérifiant la condition $P^p \subset \{A\}$ et conjugué à A^z est de la forme A^{mz}, où $m \equiv 1 \pmod p$, alors le rapport des ordres des normalisateurs des groupes $\{A^{p^{k-1}}\}$ et $\{A\}$ est congru à 1 par rapport au module p.*

Pour la démonstration des théorèmes 1—3 on applique les mêmes méthodes qui ont été suivies par l'auteur dans ses travaux précédemment cités; d'ailleurs, certaines remarques complémentaires permettent d'abréger les calculs intermédiaires. Pour déduire les théorèmes 4—5 il faut appliquer quelques méthodes nouvelles.

Il suit du théorème 5 que le symbole λ_i conserve son sens pour le cas où $p=2$, pourvu que $\mathfrak{N}_A^{(1)}=\mathfrak{N}_A^{(2)}$. Si les conditions de ce théorème sont vérifiées, nous aurons les égalités

$$\lambda_0=\lambda_1=\lambda_2=\ldots=\lambda_{k-\lambda_0}$$

et ensuite

$$\lambda_{k-\lambda_0+1}=\lambda_0-1,\ \ldots,\ \lambda_{k-2}=2,\ \lambda_{k-1}=1.$$

Ensuite on généralise d'une manière évidente un théorème démontré dans un article de W. K. Turkine et P. E. Dubuque «Sur un critère pour qu'un groupe soit non simple» (3). A savoir, il est possible en conservant la démonstration du théorème l'énoncer dans la forme suivante plus générale.

Théorème 6. *Soit A un élément d'ordre n d'un sous-groupe $\mathfrak{H}$ d'un groupe $\mathfrak{G}$; on suppose que chaque élément de $\mathfrak{H}$ conjugué à une puissance de A est encore une puissance de A. Soit $n=\prod_{i=1}^{k} p_i$, tous les p_i étant des nombres premiers, et soit $\mathfrak{N}$ le plus petit multiple des normalisateurs des groupes cycliques $\{A^{\frac{n}{p_i}}\}$. Si $\mathfrak{H}$ est un sous-groupe $\mathfrak{N}$ et si $\prod(\mathfrak{N}, A)$ n'appartient pas à la commutante $\mathfrak{H}$, le groupe $\mathfrak{G}$ possède un diviseur normal. L'ordre de ce diviseur normal est un multiple du plus grand diviseur de l'ordre du groupe $\mathfrak{G}$ premier avec l'ordre du groupe $\mathfrak{H}$.*

Le symbole $\prod(\mathfrak{N}, A)$ a le même sens que dans l'article cité (3).

En appliquant le théorème 2 ainsi que certaines propositions de l'article précédemment cité (2), on peut obtenir le critère suivant pour qu'un groupe soit non simple.

Théorème 7. *Soit A un élément d'ordre p^k d'un groupe $\mathfrak{G}$ (p étant un nombre premier impair). Soit $\mathfrak{P}$ le p-sous-groupe de Syloff du $(k-1)$-ième quasi-normalisateur de l'élément A. Si chaque élément de $\mathfrak{P}$ conjugué à A^z est de la forme A^{mz}, où $m\equiv 1 \pmod p$, et si l'élément $A^{p^{k-1}}$ n'appartient pas à la commutante du p-sous-groupe de Syloff du normalisateur de l'élément A, alors le groupe $\mathfrak{G}$ possède un diviseur normal. L'ordre de ce diviseur normal est un multiple du plus grand diviseur de l'ordre du groupe $\mathfrak{G}$ premier avec p.*

Le théorème 7 sera encore vrai dans le cas, où $p=2$, si l'on ajoute aux conditions de son énoncé la condition supplémentaire: l'élément $A^{p^{k-2}}$ ne doit pas être conjugué à son élément inverse.

Ensuite, en appliquant le théorème 6, le théorème 5, une proposition auxiliaire citée dans l'article (2) et un lemme démontré dans l'article de Dubuque et Turkine «Sur la structure des groupes simples» (4), on peut obtenir le résultat suivant.

Théorème 8. *Soit A un élément d'ordre p^k du groupe $\mathfrak{G}$ (p étant un nombre premier impair). Soit $\mathfrak{P}$ le p-sous-groupe du normalisateur du groupe cyclique $\{A\}$. Si chaque élément de $\mathfrak{P}$ conjugué à A^z est de la forme A^{mz}, où $m\equiv 1 \pmod p$, et si A n'appartient pas à la commutante $\mathfrak{P}$, le groupe $\mathfrak{G}$ possède un diviseur normal. L'ordre de ce diviseur normal est un multiple du plus grand diviseur de l'ordre du groupe $\mathfrak{G}$ premier avec p.*

Le théorème 8 est encore vrai pour le cas, où $p=2$, pourvu que $\mathfrak{N}_A^{(1)}=\mathfrak{N}_A^{(2)}$. On obtient comme corollaire des résultats précédents le théorème suivant.

Théorème 9. *Soit A un élément d'ordre p^k d'un groupe $\mathfrak{G}$ (p étant un nombre premier). Soit $\mathfrak{P}$ le p-sous-groupe de Syloff du normalisateur de l'élément A. Si aucune puissance de A n'est conjuguée à l'un des éléments de $\mathfrak{P}$ et si A n'appartient pas à la commutante $\mathfrak{P}$, alors le groupe $\mathfrak{G}$ possède un diviseur normal. L'ordre de ce diviseur normal est un multiple du plus grand diviseur de l'ordre du groupe $\mathfrak{G}$ premier avec p.*

Le théorème 9 est une généralisation d'un théorème connu de W. Burnside (5) ainsi que des théorèmes de W. H. Turkine (6) et de l'auteur

Institut Mathématique.
Université de Moscou.

Manuscrit reçu
le 27. V. 1939.

LITTÉRATURE CITÉE

[1] C. R. Acad. Sci. URSS, XXII, No. 3 (1939). [2] ibid., XXIII, No. 1 (1939). [3] ibid., XXI, No.4 (1938); ibid XX, No. 7—8 (1938). [5] W. Burnside, Theory of groups of finite order, p. 327; О. Ю. Шмидт, Абстрактная теория групп, 168 (1935). [6] Math. Annalen, III (1935).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIV, № 2

MATHÉMATIQUES

FONCTIONS DE GREEN DES ÉQUATIONS PARABOLIQUES

Par P. V. SOLOVIEV

(Présenté par S. L. Soboleff, de l'Académie, le 18. V. 1939)

Supposons que dans un espace (x, y, t) l'on se donne une région

$$\overline{D} = D \left\{ \begin{array}{l} 0 \leqslant x \leqslant b \\ 0 \leqslant y \leqslant c \\ 0 \leqslant t \leqslant h \end{array} \right\}$$

Nous donnerons le nom de parallélépipède à cette région. Il nous faut trouver une fonction $G(x, y, t; \xi, \eta, \zeta) = G(M, P)$, qui pourrait être, par rapport aux variables x, y, t, une solution de l'équation

$$\frac{\partial^2 G}{\partial x^2} + \frac{\partial^2 G}{\partial y^2} - \frac{\partial G}{\partial t} = 0 \tag{1}$$

dans la région D et, par rapport aux variables ξ, η, ζ, une solution de l'équation conjuguée à cette dernière (point $P \neq M$) dans la même région. La fonction $G(M, P)$ a dans la région $\overline{D}$ la même singularité que la fonction

$$\frac{1}{t - \xi} e^{-\frac{(x-\xi)^2 + (y-\eta)^2}{4(t-\zeta)}} \quad (\zeta \leqslant t). \tag{2}$$

A la frontière * de la région D, la fonction $G(M, P) = 0$. Dans la suite nous allons appeler cette fonction $G(M, P)$ la fonction de Green pour le parallélépipède D de l'équation (1). En conservant la méthode que nous avons suivi en formant la fonction de Green pour le trapèze de l'équation de la chaleur **, nous obtiendrons:

$$G(M, P) = \frac{1}{t - \zeta} \sum_{n=1}^{\infty} \{ e^{-\frac{[2(n-1)b + x + \xi]^2}{4(t-\zeta)}} + e^{-\frac{[2nb - x - \xi]^2}{4(t-\zeta)}} - e^{-\frac{[2(n-1)b + x - \xi]^2}{4(t-\zeta)}} - e^{-\frac{[2nb - x + \xi]^2}{4(t-\zeta)}} \} \cdot \sum_{m=1}^{\infty} \{ e^{-\frac{[2(m-1)c + y + \eta]^2}{4(t-\zeta)}} + e^{-\frac{[2mc - y - \eta]^2}{4(t-\zeta)}} - e^{-\frac{[2(m-1)c + y - \eta]^2}{4(t-\xi)}} - e^{-\frac{[2mc - y + \eta]^2}{4(t-\zeta)}} \} \equiv \frac{1}{t - \zeta} g(x, t; \xi, \zeta \mid b)\, g(y, t; \eta, \zeta \mid c). \tag{3}$$

* Sous le nom de frontière de la région D nous allons entendre les faces du parallélépipède $y = 0$, $y = c$, $x = 0$, $x = b$.

**, Voir C. R. Acad. Sci. URSS, XXIII, No. 2 (1939).

La série $g(x, t; \xi, \zeta \mid b)$ est absolument et uniformément convergente dans la région $\overline{D}$. En effet, la série convergente

$$\sum_{n=1}^{n_0-1} \{e^{-\frac{[2(n-1)b+x+\xi]^2}{4(t-\zeta)}} + e^{-\frac{[2nb-x-\xi]^2}{4(t-\zeta)}} - e^{-\frac{[2(n-1)b+x-\xi]^2}{4(t-\zeta)}} - e^{-\frac{[2nb-x+\xi]^2}{4(t-\zeta)}} +$$

$$+4 \sum_{n=n_0}^{\infty} e^{-kn^2} \tag{4}$$

est une série majorante par rapport à la série $g(x, t; \xi, \zeta \mid b)$. Ici on a choisi «$n_0$» et «$k$» de façon que

$$B_n = \min \left\{ \frac{\left[2\left(1-\frac{1}{n}\right)b+\frac{x-\xi}{n}\right]^2}{4(t-\zeta)}; \frac{\left[2b-\frac{x+\xi}{n}\right]^2}{4(t-\zeta)}; \frac{\left[2\left(1-\frac{1}{n}\right)b+\frac{x-\xi}{n}\right]^2}{4(t-\zeta)}; \right.$$

$$\left. \frac{\left[2b-\frac{x-\xi}{n}\right]^2}{4(t-\zeta)} \right\} \geqslant k > 0$$

dans le cas où $n \geqslant n_0$ (ce qui est possible). Par conséquent, la série $g(x, t; \xi, \zeta \mid b)$ est absolument et uniformément convergente dans la région $\overline{D}$. Pareillement, on peut établir bien aisément la convergence absolue et uniforme des séries

$$\frac{\partial^2 g}{\partial x^2}, \frac{\partial^2 g}{\partial \xi^2}, \frac{\partial g}{\partial t}, \frac{\partial g}{\partial \zeta}$$

obtenues de la série $g(x, t; \xi, \zeta \mid b)$ par une dérivation terme à terme. La série (3) qui représente le produit de deux séries absolument et uniformément convergentes est de même absolument et uniformément convergente dans la région $\overline{D}$ ainsi que les séries

$$\frac{\partial^2 G}{\partial x^2}, \frac{\partial^2 G}{\partial y^2}, \frac{\partial^2 G}{\partial \xi^2}, \frac{\partial^2 G}{\partial \eta^2}, \frac{\partial G}{\partial t}, \frac{\partial G}{\partial \zeta}.$$

Ensuite après avoir fait une substitution, nous allons nous convaincre que

$$G(M, P)\Big|_{\substack{\xi=0 \\ \xi=b}} \equiv G(M, P)\Big|_{\substack{\eta=0 \\ \eta=c}} \equiv 0.$$

Ainsi la série (3) représente la fonction de Green pour le parallélépipède $\overline{D}$ de l'équation (1).

A présent formons la fonction de Green $G(M, P)$ pour un parallélépipède, à $n+1$ dimensions

$$\overline{D} = D \begin{pmatrix} 0 \leqslant x_1 \leqslant a_1 \\ \cdots\cdots\cdots \\ 0 \leqslant x_n \leqslant a_n \\ 0 \leqslant t \leqslant h \end{pmatrix} \text{ de l'équation}$$

$$\Delta u \equiv \sum_{i=1}^{n} \frac{\partial^2 u}{\partial x_i^2} = \frac{\partial u}{\partial t}. \tag{5}$$

En agissant d'une manière analogue à ce qui précède, nous obtiendrons

$$G(M, P) = \frac{1}{(t-\zeta)^{\frac{n}{2}}} \prod_{i=1}^{n} g(x_i, \xi_i; t, \zeta \mid a_i) \tag{6}$$

où

$$g(x_i, \xi_i; t, \zeta \mid a_i) = \sum_{n=1}^{\infty} \{ e^{-\frac{[2(n-1)a_i + x_i + \xi_i]^2}{4(t-\zeta)}} + e^{-\frac{[2na_i - x_i - \xi_i]^2}{4(t-\zeta)}} - e^{-\frac{[2(n-1)a_i + x_i - \xi_i]^2}{4(t-\zeta)}} -$$

$$- e^{-\frac{[2na_i - x_i + \xi_i]^2}{4(t-\zeta)}} \}. \qquad (7)$$

La série (6) qui représente un produit de «n» séries absolument et uniformément convergentes est aussi une série absolument et uniformément convergente dans la région $\overline{D}$. Les séries qu'on a obtenues de la série (6) au moyen d'une dérivation terme à terme

$$\frac{\partial^2 G}{\partial x_i^2}, \quad \frac{\partial^2 G}{\partial \xi_i^2}, \quad \frac{\partial G}{\partial t}, \quad \frac{\partial G}{\partial \zeta}, \quad (i = 1, 2, \ldots n)$$

seront de même absolument et uniformément convergentes dans la région $\overline{D}$. La fonction $G(M, P)$ possède dans la région $\overline{D}$ la même singularité que la fonction

$$\frac{1}{(t-\zeta)^{\frac{n}{2}}} e^{-\frac{r^3}{4(t-\zeta)}}; \quad \left[r^2 = \sum_{i=1}^{n} (x_i = \xi_i)^2 \right].$$

Ensuite

$$G(M, P)\,|_{\xi_i = a_i} \equiv G(M, P)\,|_{\xi_i = 0} \equiv 0, \quad (i = 1, 2, \ldots n).$$

La fonction ainsi formée $G(M, P)$ représente effectivement la fonction de Green de l'équation (5) pour le parallélépipède $\overline{D}$ à $(n+1)$ dimensions.

La fonction de Green pour une pyramide à $(n+1)$ dimensions

$$\overline{D}_1 = D_1 \left\{ \begin{array}{c} k_1 t \leqslant x_1 \leqslant k_1' t + a_1 \\ k_2 t \leqslant x_2 \leqslant k_2' t + a_2 \\ \cdots\cdots\cdots\cdots \\ \cdots\cdots\cdots\cdots \\ k_n t \leqslant x_n \leqslant k_n' t + a_n \\ 0 \leqslant t \leqslant h \end{array} \right\}$$

où

$$h = \min \left\{ \frac{a_1}{2(k_1 - k_1')}; \quad \frac{a_2}{2(k_2 - k_2')}; \quad \frac{a_n}{2(k_n - k_n')} \right\}$$

aura la forme

$$G(M, P) = \frac{1}{(t-\zeta)^{\frac{n}{2}}} \prod_{i=1}^{n} g(x_i, \xi_i; t, \zeta \mid k_i, k_i', a_i)$$

où

$$g(x_i, \xi_i; t, \zeta \mid k_i, k_i', a_i) =$$

$$= \sum_{n=1}^{\infty} \{ e^{-\frac{[2nk_i t - 2(n-1)(k_i' t + a_i) - x_i - \xi_i]^2}{4(t-\zeta)} - \{ [nk_i - (n-1)k_i'] x_i - [nk_i - (n-1)k_i']^2 t + (n-1)[nk_i - (n-1)k_i'] a_i \}} -$$

$$- e^{-\frac{[2nk_i t - 2n(k_i' t + a_i) + x_i - \xi_i]^2}{4(t-\zeta)} - \{ n(k_i' - k_i) x_i - n^2 (k_i' - k_i)^2 t + n[(n+1)k_i - nk_i'] a_i \}} +$$

$$+ e^{-\frac{[2n(k_i' - t + a_i) - 2(n-1)k_i t - x_i - \xi_i]^2}{4(t-\zeta)} - \{ [nk_i' - (n-1)k_i] x_i - [nk_i' - (n-1)k_i]^2 t - n[nk_i' - (n-1)k_i] a_i \}} -$$

$$- e^{-\frac{[2nk_i t + 2n(k_i' t + a_i) + x_i - \xi_i]^2}{4(t-\zeta)} - \{ n(k_i - k_i') x_i - n^2 (k_i - k_i')^2 t + n[(n+1)k_i' - nk_i'] a_i \}} \}.$$

Université d'État d'Uzbekistan.
Samarkand

Manuscrit reçu
le 13. V. 1939

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIV, № 2

THEORY OF ELASTICITY

ON DEFORMATIONS OF BARS WITH SLIGHTLY CURVED AXIS

By P. RIZ

(Communicated by I. M. Vinogradow, Member of the Academy, 10. V. 1939)

We shall consider the deformations of a bar, the equation of whose lateral surface has the form:

$$f\left(x, y+\frac{\beta}{2}z^2\right)=0. \tag{1}$$

The parameter β which we shall consider to be small, indicates the curvature of the axis of the bar. We introduce the variables

$$\xi=x, \quad \eta=4+\frac{\beta}{2}z^2, \quad \zeta=z. \tag{2}$$

In these variables the equation of the lateral surface has the form

$$f(\xi, \eta)=0. \tag{3}$$

Observe that the following differential relations hold between the old and the new variables:

$$\frac{\partial}{\partial x}=\frac{\partial}{\partial \xi}, \quad \frac{\partial}{\partial y}=\frac{\partial}{\partial \eta}, \quad \frac{\partial}{\partial z}=\frac{\partial}{\partial \zeta}+\beta\zeta\frac{\partial}{\partial \eta}. \tag{4}$$

We shall solve here the following problems: 1) on the flexure of the bar (in the plane perpendicular to that in which lies its axis) by couples acting in its terminal cross-section, 2) on the flexure of the bar in its plane, 3) on the torsion of the bar and 4) on the tension of the bar by forces applied in its terminal cross-section. All problems are solved with the precision up to β^2. The results may be easily generalised to the case of a bar whose axis is not a plane curve, if we neglect all powers of β higher than the first.

The flexure by a concentrated force will be treated elsewhere.

1. Flexure of the Bar in the Plane *xoz* by a Couple Applied in the Terminal Cross-Section ($z=0$). We suppose the displacements to be of the form:

$$\left.\begin{aligned} u&=\frac{\alpha}{2}[\zeta^2+\nu(\xi^2-\eta^2)]+\alpha\beta u^{(1)},\\ v&=\alpha\nu\xi\eta+\alpha\beta v^{(1)},\\ w&=\alpha\xi\zeta+\alpha\beta w^{(1)}, \end{aligned}\right\} \tag{5}$$

where $u^{(1)}$, $v^{(1)}$, $w^{(1)}$ are unknown functions of ξ, η, ζ; $\alpha = -\frac{M}{EJ_\xi}$ and M is the moment of the flexing couple.

For the stresses we obtain:

$$\left.\begin{aligned} &\sigma_{xx} = \alpha\beta\sigma_{xx}^{(1)}, \quad \sigma_{xy} = \alpha\beta\sigma_{xy}^{(1)}, \\ &\sigma_{yy} = \alpha\beta\sigma_{xy}^{(1)}, \quad \sigma_{xz} = -\alpha\beta\mu\nu\eta\zeta + \alpha\beta\sigma_{xz}^{(1)}, \\ &\sigma_{zz} = -E\alpha\xi + \alpha\beta\sigma_{zz}^{(1)}, \quad \sigma_{yz} = \alpha\beta\mu\nu\xi\zeta + \alpha\beta\sigma_{yz}^{(1)} \end{aligned}\right\} \tag{6}$$

($\sigma_{xx}^{(1)}$ etc. are unknown functions of ξ, η, ζ which we shall agree to call stresses). For this system of stresses we form the equilibrium equations, the conditions expressing the absence of stresses on the lateral surface and the compatibility conditions, using the relations (4). We find

$$L_1^{(1)} - \mu\nu\eta = 0, \quad L_2^{(1)} + \mu\nu\xi = 0, \quad L_3^{(1)} = 0, \tag{7}$$

where

$$\left.\begin{aligned} &L_1^{(1)} = \frac{\partial\sigma_{xx}^{(1)}}{\partial\xi} + \frac{\partial\sigma_{xy}^{(1)}}{\partial\eta} + \frac{\partial\sigma_{xz}^{(1)}}{\partial\zeta}, \\ &M_1^{(1)} = 0, \quad M_2^{(1)} = 0, \quad M_3^{(1)} - \mu\nu\eta\zeta f_\xi' - (E - \mu\nu)\,\xi\zeta f_n' = 0, \\ &(M_1^{(1)} = \sigma_{xx}^{(1)} f_\xi' + \sigma_{xy}^{(1)} f_\eta', \ \ldots, \ M_3^{(1)} = \sigma_{xz}^{(1)} f_\xi' + \sigma_{yz}^{(1)} f_\eta') \end{aligned}\right\} \tag{8}$$

$$N_{11}^{(1)} = 0, \quad N_{(1)}^{22} = 0, \quad N_{33}^{(1)} = 0, \quad N_{12}^{(1)} = 0, \quad N_{13}^{(1)} = 0, \quad N_{23}^{(1)} = 0,$$

$$\left.\begin{aligned} &N_{11}^{(1)} = \nabla^2\sigma_{xx}^{(1)} + \frac{1}{1+\nu}\frac{\partial^2}{\partial\xi^2}(\sigma_{xx}^{(1)} + \sigma_{yy}^{(1)} + \sigma_{zz}^{(1)}), \\ &N_{12}^{(1)} = \nabla^2\sigma_{xy}^{(1)} + \frac{1}{1+\nu}\frac{\partial^2}{\partial\xi\partial\eta}(\sigma_{xx}^{(1)} + \sigma_{yy}^{(1)} + \sigma_{zz}^{(1)}), \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots \end{aligned}\right\} \tag{9}$$

From the stated conditions we obtain:

$$\left.\begin{aligned} &\sigma_{xz}^{(1)} = \mu\nu\eta\zeta, \quad \sigma_{yz}^{(1)} = (E - \mu\nu)\,\xi\zeta, \quad \sigma_{xx}^{(1)} = -\frac{E}{2}\,\xi\eta = \frac{\partial^1 F}{\partial\eta^2}, \\ &\sigma_{yy}^{(1)} = -\frac{E}{2}\,\xi\eta + \frac{\partial^2 E}{\partial\xi^2}, \quad \sigma_{xy}^{(1)} = \frac{E}{2}(\eta^2 - \xi^2) - \frac{\partial^2 F}{\partial\xi\,\partial\eta}, \\ &\sigma_{zz}^{(1)} = E\xi\eta + \nu\nabla^2 F + A\xi + B\eta + C. \end{aligned}\right\} \tag{10}$$

The function $F(\xi, \eta)$ satisfies the equation $\nabla^4 F = 0$ and the contour condition

$$\left.\begin{aligned} &F''_{\eta\eta} f_\xi' - F''_{\xi\eta} f_\eta' - \frac{E}{2}\,\xi\eta f_\xi' + \frac{E}{4}(\eta^2 - \xi^2) f_\eta' = 0, \\ &-F''_{\xi\eta} f_\xi' + F''_{\xi\xi} f_\eta' + \frac{E}{4}(\eta^2 - \xi^2) f_3' - \frac{E}{2}\,\xi\eta f_\eta' = 0. \end{aligned}\right\} \tag{11}$$

The constants A, B and C are determined from the integral conditions in the terminal cross-section:

$$\iint \sigma_{zz}\,d\xi\,d\eta = 0, \quad \iint \xi\sigma_{zz}\,d\xi\,d\eta = 0, \quad \iint \eta\sigma_{zz}\,d\xi\,d\eta = M. \tag{12}$$

To the obtained «stresses» correspond the following «displacements»:

$$\left.\begin{aligned} &u^{(1)} = -\frac{\eta\xi^2}{2} + \bar{u}(\xi, \eta, A, B, C), \\ &v^{(1)} = \frac{\xi\zeta^2}{2} + \bar{v}(\xi, \eta, A, B, C), \\ &w^{(1)} = (1+\nu)\,\xi\eta\zeta + \bar{w}(\xi, \eta, A, B, C. \end{aligned}\right\} \tag{13}$$

The functions $\bar{u}, \bar{v}, \bar{w}$ determine a certain plane deformation and are expressed through the function F. The remaining terms in the additional displacements determine a certain twist of the cross-sections accompanied by their distortion, which is proportional to z.

The relation between the curvature and the flexing moment remains the usual and the torsion (τ) caused by the flexure is determined by the formula

$$\tau = \frac{M}{EJ_\xi}\beta z. \tag{14}$$

2. **Flexure of the Bar in its Plane.** We assume the equation of the lateral surface to be

$$f\left(x + \frac{\beta}{2} z^2, y\right) = 0. \tag{15}$$

Further, we put

$$\xi = x + \frac{\beta}{2} z^2\ \eta = y,\ \zeta = z. \tag{16}$$

We note the equations:

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial \xi},\ \frac{\partial}{\partial y} = \frac{\partial}{\partial \eta},\ \frac{\partial}{\partial z} = \frac{\partial}{\partial \zeta} + \beta\zeta \frac{\partial}{\partial \xi}. \tag{17}$$

For the displacements we retain the expressions (5). With the help of the formulae (17) we determine the stresses:

$$\left.\begin{aligned} &\sigma_{xx} = -\alpha\beta\lambda\zeta^2 + \alpha\beta\sigma_{xx}^{(1)},\quad \sigma_{yy} = -\alpha\beta\lambda\zeta^2 + \alpha\beta\sigma_{yy}^{(1)},\\ &\sigma_{zz} = -\alpha E\xi - (\lambda + 2\mu)\,\alpha\beta\zeta^2 + \alpha\beta\sigma_{zz}^{(1)},\quad \sigma_{xy} = \alpha\beta\sigma_{xy}^{(1)},\\ &\sigma_{xz} = \alpha\beta\mu\nu\xi\zeta^2 + \alpha\beta\sigma_{xz}^{(1)},\quad \sigma_{yz} = \alpha\beta\mu\nu\eta\zeta^2 + \alpha\beta\sigma_{yz}^{(1)}. \end{aligned}\right\} \tag{18}$$

The equilibrium equations are:

$$L_1^{(1)} + \mu\nu\xi = 0,\quad L_2^{(1)} + \mu\nu\eta = 0,\quad L_3^{(1)} - (2\lambda + 6\mu)\,\zeta = 0. \tag{19}$$

The conditions on the lateral surface are:

$$M_1^{(1)} - \lambda\zeta^2 f_\xi' = 0,\quad M_2^{(1)} - \lambda\zeta^2 f_\eta' = 0,\quad M_3^{(1)} + \mu\nu\zeta\,(\xi f_\eta' - \eta f_\xi') = 0. \tag{20}$$

The compatibility conditions are:

$$\left.\begin{aligned} &N_{11}^{(1)} = \varkappa_1,\quad N_{12}^{(1)} = 0,\\ &N_{22}^{(1)} = \varkappa_1,\quad N_{13}^{(1)} = 0,\\ &N_{33}^{(1)} = \varkappa_2,\quad N_{23}^{(1)} = 0,\\ &\varkappa_1 = 2\lambda,\quad \varkappa_2 = \frac{2\mu}{1-2\nu}\,(6 - 5\nu - 3\nu^2). \end{aligned}\right\} \tag{21}$$

From the conditions (19)—(21) we find:

$$\left.\begin{aligned} &\sigma_{xz}^{(1)} = -\mu\nu\xi\zeta,\quad \sigma_{yz}^{(1)} = -\mu\nu\eta\zeta,\quad \sigma_{xx}^{(1)} = \lambda\zeta^2 + \frac{\partial^2 F}{\partial y^2},\quad \sigma_{yy}^{(1)} = \lambda\zeta^2 + \frac{\partial^2 F}{\partial \xi^2},\\ &\sigma_{xy}^{(1)} = -\frac{\partial^2 F}{\partial \xi\,\partial \eta},\\ &\sigma_{zz}^{(1)} = \nu\nabla^2 F + (\lambda + 6\mu + \mu\nu)\,\zeta^2 + m\,(\xi^2 + \eta^2) + A\xi + B\eta + C. \end{aligned}\right\} \tag{22}$$

The function $F(\xi, \eta)$ satisfies the equation $\nabla^4 F = n$ inside the contour $f(\xi, \eta) = 0$ and vanishes on the contour,

$$m = -\frac{\lambda\nu}{2},\quad n = \frac{\mu\nu^2}{1+\nu},$$

A, B and C are determined from the conditions in the terminal cross-section ($z=0$):

$$\iint \sigma_{zz}\, d\xi d\eta = 0, \quad \iint \xi \sigma_{zz}\, d\xi d\eta = M, \quad \iint \eta \sigma_{zz}\, d\xi d\eta = 0. \tag{23}$$

We shall not state here the displacements which amount to a certain sloping and distortion of the cross-sections, and only point out that the usual relation between the increment of the curvature and the flexing moment is preserved here, too.

Combining the solutions of the problems 1) and 2), we may determine the deformations of the bar, the lateral surface of which is given by the equation

$$f\left(x+\beta_1 \frac{z^2}{2},\ y+\beta_2 \frac{z^2}{2}\right)=0. \tag{24}$$

This may be obviously done, if we retain the linearity of the problem, i. e. if we neglect the products and the squares of the parameters β_1 and β_2.

The case of an oblique flexure of a plane bar may be, of course, also obtained by superposition of the solutions of the problems 1) and 2).

The method is applicable also to bars with the lateral surface determined by the equation

$$f(x+\beta_1 z^{n_1},\quad y+\beta_2 z^{n_2})=0,$$

where n_1 and n_2 are integers, but in this case the solution must be looked for in the form of polynomials in z.

Received
13.V.1939.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIV, № 2

MÉCANIQUE CÉLESTE

SUR LA SIMPLIFICATION DE L'ÉQUATION DE LA TRAJECTOIRE DANS LE PROBLÈME DE DEUX CENTRES FIXES

Par G. BADALIAN

(Présenté par V. G. Fessenkov, de l'Académie, le 14. V. 1939)

L'équation de la trajectoire dans le problème de deux centres fixes peut être écrite sous la forme suivante:

$$I(\lambda) = I_1(\mu) + \beta \tag{1}$$

où

$$I(\lambda) = \int \frac{d\lambda}{\sqrt{(\lambda^2-1)\,R(\lambda)}} \qquad I_1(\mu) = \int \frac{d\mu}{\sqrt{(\mu^2-1)\,S(\mu)}} \tag{2}$$

et

$$R(\lambda) = h\lambda^2 + k^2\lambda + \alpha, \qquad S(\mu) = h\mu^2 + k_1^2\mu + \alpha \tag{3}$$

où λ et μ sont des coordonnées elliptiques du point mobile. Les formules représentant ces coordonnées comme des fonctions des coordonnées cartésiennes, aussi bien que tous les autres éclaircissements nécessaires on trouve dans la littérature sur la question ([1], [2], [3], [4]). Les désignations de la Note présente sont les désignations des travaux cités ([3], [4]). En vertu du choix des coordonnées et du système des unités employés les variables λ et μ satisfont toujours aux inégalités suivantes

$$1 \leqslant \lambda \leqslant \infty, \qquad -1 \leqslant \mu \leqslant 1. \tag{4}$$

Dans les cas spéciaux des types du mouvement particuliers les intervalles (4) sont à remplacer par d'autres intervalles plus étroits, indiqués par nous dans ce qui va suivre. Le problème principale de la Note présente consiste dans la recherche des substitutions de la forme

$$\lambda = \frac{a_0 + a_1\xi + a_2\xi^2}{b_0 + b_1\xi + b_2\xi^2}, \qquad \mu = \frac{\alpha_0 + \alpha_1\eta + \alpha_2\eta^2}{\beta_0 + \beta_1\eta + \beta\eta^2} \tag{5}$$

à l'aide desquelles les intégrales elliptiques (2), entrant dans la formule (1), peuvent être réduites à la forme normale de Legendre:

$$I(\lambda) = L\int \frac{d\xi}{\sqrt{(1-\xi^2)(1-\varkappa^2\xi^2)}} \qquad I_1(\mu) = M\int \frac{d\eta}{\sqrt{(1-\eta^2)(1-\nu^2\eta^2)}}. \tag{6}$$

La recherche de substitutions d'un tel genre semble présenter un intérêt considérable pour le but de la théorie quantitative du mouvement dans le problème indiqué, parce qu'on peut deviner que les substitutions de la forme (5) nous fourniront des formules deréduction qui seront plus simples que les formules, résultant des substitutions linéaires, étudiées par M. Tallquist ([2]). Il est à remarquer aussi que nous nous posons

Table sommaire des substitutions pour la variable λ — Table I

Type du mouvement	Substitutions pour λ	Valeurs du coefficient L	Intervalles du changement de ξ	Valeurs de module $\varkappa$
127, 123, 724; $h<0$ $\lambda_1<1<\lambda_2$ $1\leqslant\lambda\leqslant\lambda_2$	1) $\lambda=\dfrac{2\lambda_2-(\lambda_2-1)\xi^2}{2+(\lambda_2-1)\xi^2}$ 2) $\lambda=\dfrac{\lambda_2-\lambda_1-\lambda_1(\lambda_2-1)\xi^2}{\lambda_1-1-(\lambda_2-1)\xi^2}$	1) $L=\dfrac{-\sqrt{2}}{\sqrt{h(\lambda_1-\lambda_2)}}$ 2) $L=\dfrac{\sqrt{2}}{\sqrt{h(\lambda_1-\lambda_2)}}$	1) $1\geqslant\xi^2\geqslant 0$ 2) $0\leqslant\xi^2\leqslant 1$	$\varkappa^2=\dfrac{(\lambda_1+1)(\lambda_2-1)}{2(\lambda_2-\lambda_1)}$
131; $h<0$ $1<\lambda_1<\lambda_2$ $\lambda_1\leqslant\lambda\leqslant\lambda_2$	1) $\lambda=\dfrac{\lambda_2(\lambda_1+1)-(\lambda_2-\lambda_1)\xi^2}{\lambda_1+1+(\lambda_2-\lambda_1)\xi^2}$ 2) $\lambda=\dfrac{\lambda_1(\lambda_2-1)-(\lambda_2-\lambda_1)\xi^2}{\lambda_2-1-(\lambda_2-\lambda_1)\xi^2}$	1) $L=\dfrac{-2}{\sqrt{h(1+\lambda_1)(1-\lambda_2)}}$ 2) $L=\dfrac{2}{\sqrt{h(1+\lambda_1)(1-\lambda_2)}}$	1) $1\geqslant\xi^2\geqslant 0$ 2) $0\leqslant\xi^2\leqslant 1$	$\varkappa^2=\dfrac{2(\lambda_2-\lambda_1)}{(\lambda_1+1)(\lambda_2-1)}$
214 (214a); $h>0$ $-1<\lambda_2<\lambda_1<1$ $1\leqslant\lambda\leqslant\infty$	1) $\lambda=\dfrac{2\lambda_2\xi^2+1-\lambda_2}{2\xi^2-1+\lambda_2}$ 2) $\lambda=\dfrac{2\lambda_1\xi^2-1-\lambda_1}{2\xi^2-1-\mu_1}$	1) $L=\dfrac{-2}{\sqrt{h(1+\lambda_1)(1-\lambda_2)}}$ 2) $L=\dfrac{2}{\sqrt{h(1+\lambda_1)(1-\lambda_2)}}$	1) $1\geqslant\xi^2\geqslant\dfrac{1-\lambda_2}{2}>0$ 2) $0\leqslant\xi^2\leqslant\dfrac{1+\lambda_1}{2}<1$	$\varkappa^2=\dfrac{2(\lambda_1-\lambda_2}{(1+\lambda_1)(1-\lambda_2)}$
211, 213, 214b; $h>0$ $\lambda_2<-1<\lambda_1<1$ $1\leqslant\lambda\leqslant\infty$	mêmes formules que dans le cas 121		1) $1\geqslant\xi^2\geqslant\dfrac{2}{1-\lambda_2}>0$ 2) $0\leqslant\xi^2\leqslant\dfrac{\lambda_1-\lambda_2}{1-\lambda_2}<1$	$\varkappa^2=\dfrac{(1+\lambda_1)(1-\lambda_2)}{2(\lambda_1-\lambda_2)}$
222; $h>0$ $\lambda_2<-1<1<\lambda_1$ $\lambda_1\leqslant\lambda\leqslant\infty$	mêmes formules que dans le cas 131		1) $1\geqslant\xi^2\geqslant\dfrac{\lambda_1+\lambda}{\lambda_1-\lambda_2}>0$ 2) $0\leqslant\xi^2\leqslant\dfrac{1-\lambda_2}{\lambda_1-\lambda_2}<1$	$\varkappa^2=\dfrac{2(\lambda_1-\lambda_2)}{(1+\lambda_1)(1-\lambda_2)}$

Table 2

Table sommaire des substitutions pour la variable μ

Type du mouvement	Substitutions pour μ	Valeurs du coefficient M	Intervalles du changement de η	Valeurs de module ν
121, 131; $h<0$ μ_1 et μ_2 sont imaginaires ou ils sont tous les deux >1 $-1\leqslant\mu\leqslant 1$	1) $\mu=\dfrac{2\mu_2\eta^2-\mu_2+1}{2\eta^2+\mu_2-1}$ 2) $\mu=\dfrac{\mu_1+1-2\mu_1\eta^2}{\mu_1+1-2\eta^2}$	1) $M=\dfrac{2}{\sqrt{h\,(1+\mu_1)\,(1-\mu_2)}}$ 2) $M=\dfrac{-2}{\sqrt{h\,(1+\mu_1)\,(1-\mu_2)}}$	1) $0\leqslant\eta^2\leqslant 1$ 2) $1\geqslant\eta^2\geqslant 0$	$\nu^2=\dfrac{2\,(\mu_2-\mu_1)}{(\mu_1+1)\,(\mu_2-1)}$
123; $h<0$ $-1<\mu_1<1<\mu_2$ 213; $h>0$ $\mu_2<-1<\mu_1<1$ $-1\leqslant\mu\leqslant\mu_1$	1) $\mu=\dfrac{\mu_2\,(1+\mu_1)\,\eta^2-\mu_2+\mu_1}{(1+\mu_1)\,\eta^2+\mu_2-\mu_1}$ 2) $\mu=\dfrac{2\mu_1-(1+\mu_1)\,\eta^2}{2-(1+\mu_1)\,\eta^2}$	1) $M=\dfrac{\sqrt{2}}{\sqrt{h\,(\mu_1-\mu_2)}}$ 2) $M=\dfrac{-\sqrt{2}}{\sqrt{h\,(\mu_1-\mu_2)}}$	1) $0\leqslant\eta^2\leqslant 1$ 2) $1\geqslant\eta^2\geqslant 0$	$\nu^2=\dfrac{(\mu_1+1)\,(\mu_2-1)}{2\,(\mu_2-\mu_1)}$
124 (124α); $h<0$ $-1<\mu_1<\mu_2<1$ $-1\leqslant\mu\leqslant\mu_1$	1) $\mu=\dfrac{(1+\mu_1)\,\eta^2-1+\mu_1}{(1+\mu_1)\,\eta^2+1-\mu_1}$ 2) $\mu=\dfrac{\mu_1\,(1+\mu_2)-\mu_2\,(1+\mu_1)\,\eta^2}{1+\mu_2-(1+\mu_1)\,\eta^2}$	1) $M=\dfrac{2}{\sqrt{h\,(\mu_1-1)\,(\mu_2+1)}}$ 2) $M=\dfrac{-2}{\sqrt{h\,(\mu_1-1)\,\mu_2+1)}}$	1) $0\leqslant\eta^2\leqslant 1$ 2) $1\geqslant\eta^2\geqslant 0$	$\nu^2=\dfrac{(1+\mu_1)\,(1-\mu_2)}{(1-\mu_1)\,(1+\mu_2)}$
124 (124β); $h<0$ $-1<\mu_1<\mu_2<1$ $\mu_2\leqslant\mu\leqslant 1$	1] $\mu=\dfrac{1+\mu_2-(1-\mu_2)\,\eta^2}{1+\mu_2+(1-\mu_2)\,\eta^2}$ 2) $\mu=\dfrac{\mu_2\,(1-\mu_1)-\mu_1\,(1-\mu_2)\,\eta^2}{1-\mu_1-(1-\mu_2)\,\eta^2}$	1) $M=\dfrac{-2}{\sqrt{h\,(\mu_1-1)\,(\mu_2+1)}}$ 2) $M=\dfrac{2}{\sqrt{h\,(\mu_1-1)\,(\mu_2+1)}}$	1) $1\geqslant\eta^2\geqslant 0$ 2) $0\leqslant\eta^2\leqslant 1$	$\nu^2=\dfrac{(1+\mu_1(\,(1-\mu_2)}{(1-\mu_1)\,(1+\mu_2)}$
214; $h>0$ $-1<\mu_2<\mu_1<1$ $\mu_2\leqslant\mu\leqslant\mu_1$	1) $\mu=\dfrac{\mu_2\,(1+\mu_1)+(\mu_1-\mu_2)\,\eta^2}{1+\mu_1-(\mu_1-\mu_2)\,\eta^2}$ 2) $\mu=\dfrac{\mu_1\,(1-\mu_2)-(\mu_1-\mu_2)\,\eta^2}{1-\mu_2-(\mu_1-\mu_2)\,\eta^2}$	1) $M=\dfrac{2}{\sqrt{h\,(1+\mu_1)\,(1-\mu_2)}}$ 2) $M=\dfrac{-2}{\sqrt{h\,(1+\mu_1)\,(1-\mu_2)}}$	1) $0\leqslant\eta^2\leqslant 1$ 2) $1\geqslant\eta^2\geqslant 0$	$\nu^2=\dfrac{2\,(\mu_1-\mu_2)}{(1+\mu_1)\,(1-\mu_2)}$
212, 213; $h>0$ $\mu_2<-1<1<\mu_1$	mêmes formules que dans le cas 121			

le problème de la recherche de la correspondance entre les types spéciaux des substitutions et les types spéciaux du mouvement.

Quant à la transformation même, il est plus commode de l'effectuer d'une manière suivante. Premièrement, nous réduisons les intégrales (2) à la forme

$$I(\lambda) = \int \frac{du}{\sqrt{H(1-Au^2)(1-Bu^2)}} \quad I_1(\mu) = \int \frac{dv}{\sqrt{H_1(1-A_1v^2)(1-B_1v^2)}} \qquad (7)$$

en faisant usage des substitutions suivantes

$$\lambda = \frac{1+u^2}{1-u^2}, \quad \mu = \frac{1+v^2}{1-v^2}, \quad \lambda = \frac{\lambda_1+\lambda_2 u^2}{1+u^2}, \quad \mu = \frac{\mu_1+\mu_2 v^2}{1+v^2} \qquad (8)$$

n'étant que les formes spéciales des substitutions (5). Les symboles λ_1, λ_2 et μ_1, μ_2 désignent dans les formules (8) les racines des polynômes $R(\lambda)$ et $S(\mu)$ et H, A, B et H_1, A_1, B_1 sont des fonctions de ces racines λ_1, λ_2 et μ_1, μ_2, lesquelles à son tour sont des fonctions bien déterminées des constantes de l'intégration h et α.

D'après cette transformation préliminaire nous pouvons faire usage de la théorie usuelle de la réduction des intégrales elliptiques (7) à la forme normale de Legendre. Cette théorie [5] nous indique des substitutions

$$u^2 = \frac{a_0' + a_2'\xi^2}{b_0' + b_2'\xi^2} \quad \text{et} \quad v^2 = \frac{\alpha_0' + \alpha_2'\eta^2}{\beta_0' + \beta_2'\eta^2} \qquad (9)$$

où l'un des coefficients peut être posé égal à 1, et où l'un, au moins, des coefficients (et dans certains cas les deux des coefficients) doit être égal à zéro, et où de plus les deux coefficients (ou bien un coefficient) qui sont restés libres sont des fonctions bien déterminées des constantes de l'intégration. Dans le cas où un seul des coefficients dépend des constantes de l'intégration α et h, le deuxième est égal à 1 ou bien à zéro.

Les substitutions (9) sont des substitutions finales, achevant le procédé de la réduction à la forme normale.

Dans nos deux tables sommaires à l'aide desquelles nous résumons nos résultats, nous n'indiquons ni la substitution (8), ni la substitution (9), mais la substitution définitive, résultant des formules (8) et (9) d'après l'élimination des variables auxiliaires u et v. Les substitutions définitives nommées sont de la forme générale (5)

$$\lambda = \frac{a_0 + a_2\xi^2}{b_0 + b_2\xi^2}, \quad \mu = \frac{\alpha_0 + \alpha_2\eta^2}{\beta_0 + \beta_2\eta^2}, \qquad (10)$$

n'étant que les formes particulières de la substitution (5).

Comme on voit dans les tables, nous avons disposé les diverses substitutions en indiquant les types du mouvement correspondants, désignés par les symboles décimales, introduits par nous dans notre article cité. A chaque type de mouvement correspond non une seule, mais deux substitutions différentes: deux pour la variable λ et deux pour la variable μ. Une de ces deux substitutions est un peu plus compliquée que l'autre. Quant à l'expression du module de l'intégrale élliptique, elle est la même dans les deux cas.

Il faut noter enfin que la correspondance qui existe entre les types du mouvement, d'un part, et les types des substitutions, de l'autre, n'est pas une correspondance biunivoque.

Manuscrit reçu
le 19. V. 1939.

LITTÉRATURE CITÉE

[1] L. Charlier, Die Mechanik des Himmels, I. [2] Acta Soc. Scient. Fennicae, Nov. Ser. A. No. 1 (1927). [3] Астроном. журн., XI, вып. 4 (1934). [4] Comment. Physico-Math. Soc. Scient. Fennicae, VIII, No. 2 (1935). [5] Montessus de Ballore, Leçons sur les fonctions elliptiques.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIV, № 2

PHYSICS

DIVISION OF URANIUM NUCLEI UNDER THE ACTION OF NEUTRONS AND THE PROBLEM OF THE EXISTENCE OF TRANS-URANIUMS

By V. G. KHLOPIN, Member of the Academy, M. A. PASVICK-KHLOPINA and N. F. VOLKOV

Shortly upon having discovered the artificial radioactivity of uranium under neutron bombardment (1934), E. Fermi (1) became aware that two of the artificial products of transmutation obtained, having half- lives of 13 and 90 min. respectively, differed markedly in respect of their chemical properties from the elements located between numbers 82 and 92 in the Periodic System, wherefore they should be assigned atomic numbers higher than 92. Fermi's suggestion as to the formation out of uranium of chemical elements which would lie in the Periodic System beyond uranium and further on known under the general name of trans-uraniums, encountered severe criticism on the part of A. Grosse (2) and I. Noddak (3). However, in later works by O. Hahn and L. Meitner (4) E. Fermi (5) and the authors of the present paper (6) furnished incontrovertible evidence in support of Fermi's main data by closer investigation of the chemical properties of the two new artificial radioelements. In the subsequent, the problem of the existence of trans-uraniums and of their chemical properties was considered in detail in a number of brilliant works by O. Hahn and L. Meitner (7); and the data obtained by these authors received in their major part full evidential support from Curie and Savitch (8). The fact of the existence of trans-uraniums thus seemed to be set upon a firm foundation. It is true that later on an article published by Curie and Savitch (9) in 1938 brought some dissent into the harmonious scheme of successive transmutations suggested for uranium by O. Hahn and L. Meitner. Curie and Savitch have succeeded in identifying among the products of the artificial transmutation of uranium a radioelement with $T=3.5$ h. which element came in respect of its chemical properties very close to lanthanum and was originally mistaken by the authors for an isotope of actinium. Thus the matter stood until in December 1938 Hahn and Strassmann (10) published an article in which they reported the discovery of isotopes of radium and actinium among the products of artificial transmutation of uranium acted upon with neutrons. In a subsequent paper by the same authors (11) (January 1939) there was given a strict chemical demonstration of that the products of uranium transmutation which they had originally mistaken for the isotopes of radium and actinium were, in point of fact, the isotopes of barium and lanthanum, the latter being a pro-

duct of the transmutation of the former. On the basis of the data obtained O. Hahn assumed the existence of a new type of nuclear decay in uranium which involves the fission of the nucleus into two fragments, under formation of elements lying far beyond uranium in the Periodic System. Independently of O. Hahn, somewhat later, F. Joliot ([12]) showed the theoretical probability of the fission of the uranium and thorium nuclei under neutron bombardment into two fragments of a closely approaching mass, giving at the same time a most elegant and simple physical proof of the actual occurrence of this process. Simultaneously and independently the same results were reached by Meitner and Frisch ([13]) and somewhat later by Frisch ([14]) who suggested experimental evidence speaking in favour of the existence of fragments of uranium nucleus with great energy.

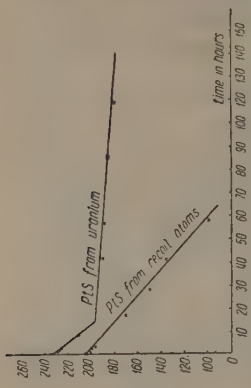
The discovery of a new type of uranium disintegration under neutron bombardment, in the course of which may be formed nuclei of atoms of lighter elements, to be located in the center of Mendeleyeff's table, made questionable the very fact of existence of chemical elements with a greater atomic number than uranium. In view of the outstanding importance which belongs in chemistry to the problem of the existence of «trans-uraniums», we deemed it of interest to attempt solution of this controversial question with the help of the cyclotron of the Radium Institute which offered the possibility to obtain a powerful source of neutrons according to the reaction ${}_1^2D + {}_1^2D \rightarrow {}_2^3\mathrm{He} + {}_0^1n$.

In doing so, we started from the following considerations: all the products of the decay of an uranium nucleus obtained through the fission of the nucleus of this element after the capture of a neutron, must (since they are formed with a considerable gain in energy) possess at the moment of their formation a great kinetic energy which makes them to be repulsed far off from the spot of their formation. It was this very circumstance that enabled F. Joliot ([12]) to make use of the «method of radioactive recoil» for gathering them. On the contrary, the «nuclei of trans-uraniums», if likely to form at all, will be heavier than uranium nuclei formed through the loss of a β-particle by the newly formed compound nucleus of the uranium isotope and, accordingly, will carry no appreciable kinetic energy, thus always remaining at the spot of their formation, i. e. among the surrounding atoms of uranium. Consequently, if «trans-uraniums» do actually exist, we never will find them among the recoil atoms; indeed, the curves of decay obtained for the products of artificial transmutation of uranium emitted directly by uranium under neutron bombardment, should differ from those obtained for the decay of recoil atoms formed in the course of this reaction. To check this assumption, the following experiments were carried out. We have bombarded simultaneously thin layers of uranium oxide (U_3O_8) (applied on wood and covered with a glass plate 3 mm away) and sodium uranate with neutrons emitted from the cyclotron. The intensity of neutrons produced corresponded to about 30 Curie Rn+Be. The exposure varied from a few to 36 hours.

The bombarded sodium uranate and the recoil atoms collected on the glass plates were then subjected to the same chemical treatment which consisted in the following. First of all the two materials were dissolved in hydrochloric acid whose concentration was brought to 7—8%; 10—20 mg of La $(NO_3)_3$ were added to the solution, as the same amount of potassium chloroplatinate, whereupon the solution preliminarily heated to boiling point was in both cases precipitated with hydrogen sulphide. The PtS formed was filtered off, washed, first with dilute hydrochloric acid and then with water, and in the case of recoil atoms was subjected to measurement directly upon drying. Contrariwise, in the case of sodium uranate, in order to totally eliminate UX_1, the formed precipitate of PtS was filtered off through a Schott

glass filter, dissolved in aqua regia, after which the filtrate was twice evaporated with hydrochloric acid, whereupon the hydrochloric acid solution after the addition of 10—15 mg of lanthanum salt was again precipitated with hydrogen sulphide. The newly obtained precipitate PtS_2 was washed with dilute hydrochloric acid and water, dried and then measured. Measurements were effected with the help of a duraluminium Geiger-Müller electron-counter with a wall thickness of 0. 12 mm. The PtS_2 precipitate dried on the filter was wrapped in a thin sheet of cellophane and next wound round the counter.

The chemical treatment after O. Hahn and L. Meitner adopted by us upon repeated checks, is considered to be the most efficient for obtaining trans-uraniums in the pure state. The curves for decay of platinum sulphide obtained in the course of this procedure were subjected to the most careful observation during a period of time extending to 2 or 3 weeks. We took several photographs of parallel curves of decay. While showing at the beginning, i. e. for the first 15—20 hours, an almost identical drop, these parallel curves next, i. e. within the period following the 15—20 hours, inevitably exhibited a most different course of the disintegration process. Whereas the curve for platinum sulphide from recoil atoms showed a total decay already after 30—40 hours, that for platinum sulphide from sodium uranate showed already after 15—20 hours a break, corresponding to the formation of a more stable radioelement with a half-life of about 70 hours which, according to L. Meitner and O. Frisch ([15]), corresponds to a typical trans-uranium eka Ir with $T=66$ h. A pattern of the curves of decay for platinum sulphide obtained from recoil atoms and from sodium uranate is given in Fig. 1.



Фиг. 1.

It would seem that the results obtained by us might give tangible proof of the existence of trans-uraniums. However, the experimental works by L. Meitner and O. Frisch ([15]), on the one hand, and by Bretscher and Cook ([16]), on the other, published within the last two months, seem to permit of giving a double interpretation to the results obtained by us, since these authors conducted analogous experiments and obtained quite opposite results. Thus, L. Meitner and O. Frisch, while observing the decay of platinum sulphide obtained from recoil atoms and uranium, found the disintegration curves to run entirely parallel during the first 20 to 25 hours. Bretcher and Cook subjected uranium to a lengthy bombardment and collected the recoil atoms on a glass plate 1 mm away from the uranium surface. In the platinum sulphide extracted thereupon they found the presence of an element with $T=70$—72 h. The only difference in the procedure we used and that employed by Meitner and Frisch and Bretscher and Cook consisted in the fact that the latter collected the recoil atoms at a much closer distance from the bombarded uranium than we did (1 mm instead of 3 mm in our case). In order to explain the discrepancy between the results obtained by us, on the one hand, and by Meitner and Frisch and by Bretscher and Cook, on the other, the following two assumptions can be made:

1. At such a close distance of the collecting plate from uranium as chosen by the above authors, the plate may collect besides recoil atoms proper, also uranium atoms mechanically entrained from surface layers, as well as newly formed trans-uranium atoms. In this case, however, the activity

of the trans-uranium atoms deposited on the plate should be much less than that of the typical recoil atoms.

2. Along with the firmly established type of fission of the uranium nucleus into two fragments which is accompanied by a great gain in energy, there may take place also another type of fission which is less advantageous from the energy viewpoint. In this case the products of decay do not show great kinetic energy and are repulsed only to a short distance from the spot of their formation.

So far the experimental data obtained do not permit of deciding between the two alternatives, which requires further experiments. Investigation is in progress.

Radium Institute.
Academy of Sciences of the USSR.

Received
9. V. 1939.

REFERENCES

[1] E. Fermi, Nature, **133**, 898 (1934). [2] A. Grosse a. M. S. Agruss, Nature, **134**, 773 (1934). [3] I. Noddack, ZS. f. angew. Chem., **47**, 301 (1934). [4] O. Hahn u. L. Meitner, Naturw., **23**, 37, 230, 544 (1935). [5] E. Fermi, E. Amaldi, O. d'Agostino, F. Rasetti a. E. Segré, Proc. Roy. Soc. (A), **146**, 483 (1934); **149**, 522 (1935). [6] М. А. Пасвик-Хлопина и В. Г. Хлопин, Сб. к 50-летию науч. и педаг. деят. акад. В. И. Вернадского, 539 (1936). [7] O. Hahn, L. Meitner u. F. Strassmann, Ber., **69**, 905 (1935); L. Meitner, O. Hahn u. F. Strassmann, ZS. f. Physik, **106**, 249 (1937); **109**, 538 (1938). [8] I. Curie et P. Savitch, Journ. de Phys. et le Radium, VIII, 385 (1937). [9] I. Curie et P. Savitch, ibid., **9**, 335 (1938). [10] O. Hahn u. F. Strassmann, Naturw., **26**, 755 (1939). [11] O. Hahn u. F. Strassmann, ibid, **27**, 11 (1939). [12] F. Joliot, C. R., **208**, 341 (1939). [13] L. Meitner a. O. Frisch, Nature, **143**, 246 (1939). [14] O. Frisch, ibid., **143**, 276 (1939). [15] L. Meitner a. O. Frisch, ibid., **143**, 471 (1939). [16] E. Bretscher a. L. Cook, ibid., **143**, 559 (1939).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIV, № 2

PHYSICS

ON THE NATURE OF THE ANOMALOUS PHYSICAL PROPERTIES OF THIN SILVER FILMS

By A. T. ASHTCHEULOV

(Communicated by S. I. Wawilow, Member of the Academy, 6. V. 1939)

In thin metallic layers applied to transparent substrata, a sharp decline in electroconductivity and change of optical constants n and k takes place upon reduction of the thickness of the layer below a certain critical value (of the order of several mμ).

At the present time the generally accepted point of view is that the anomalous physical properties of thin metallic layers are due to their structure being different from the structure of metals in massive condition. Two hypotheses are advanced for the case in hand: 1) that of the colloidal and 2) that of the amorphous structure of a metallic layer*.

We have made an attempt to throw some light on the nature of these anomalous properties of thin layers, taking our starting point in observations of light absorption by films of silver and some other metals.

The choice of an optical method for the solution of a problem which is essentially a structural one, there being available direct methods for the determination of the structure of thin layer, was made on account of the following considerations:

1. The optical properties of colloidal metals in both the liquid and the solid phase have been experimentally studied rather in detail. Moreover, theoretical conceptions (Rayleigh-Mi's theory) have also been developed in this field, which permit of predicting the optical properties of such systems and inversely, to judge of their structure according to the absorption of light. M. V. Savostianova (2) has shown in a number of works that the observed optical properties of crystals containing colloidal metals agree in a satisfactory manner with Rayleigh-Mi's theory. Therefore, with the availability of observations testifying to the colloidal structure of thin metallic layers, it appears fully lawful to apply to their study the mechanisms of the optics of colloidal metals**.

2. There are good reasons to assume that the structure of the thinnest layers is in the very beginning of their life fairly unstable. Therefore, it is preferable in our case to make use of such methods of investigation which are safe from the risk of breaking up in the course of measurements the struc-

* A more detailed account of this can be found in a reviewing article of the author(1).

** For results of work along these lines see articles (3,4).

ture of the substance under investigation. The optical method seems to be faultless in this respect.

Layers of silver were applied under high vacuum to substrata made of glass, fused quartz and crystals of NaCl and KCl. Silver in the form of a loop was suspended on a tungsten wire and volatilized by heating it with current. In course of the condensation of silver vapour on the substrata their temperature could be varied at will within the broadest range. Experimental conditions usual with work of this kind were established, insuring freedom from contamination of the metal layers (use of liquid air, outgasing, etc.). Measurements of light absorption by silver films were effected with a Hilger quartz spectrograph provided with Specker and J. Lewis's photometric attachments, use being made of glass monochromators of different systems.

We measured the absorption of light by silver films according to the thickness of the layer and the degree of heating of the substratum during the process of dusting-on. The results obtained are as follows:

1. Layers obtained with the substratum at room temperature exhibit a selective absorption of light depending upon the thickness of the silver layer. Thus, the thinnest layers—of a thickness of the order of 1 mμ—appear yellow in passing light, and their maximum absorption is found to lie approximately at 430—440 mμ; with increase in the thickness of the layer, the absorption maximum becomes displaced in the direction of long wave lengths and at 6—7 mμ it lies already within the red region of the spectrum (in passing light such layers appear blue). Further increase of thickness fails to produce any visible change in colour. The course of the process of light absorption by thin layers of silver on glass is represented in Fig. 1.

2. A radically different picture is observable in case if the substratum is heated during the process of dusting-on. By means of such procedure it is possible to obtain sufficiently thick layers with anomalous colouring. Thus, at a temperature of the substratum of 300—350° C and with appropriate velocity of dusting-on, layers of 10—15 mμ in thickness exhibit anomalous optical properties, with absorption maxima lying approximately at the same spot as absorption maximum for the thinnest layers. A displacement of anomalous optic properties was noticeable with all substrata used. Krautkrämer* has shown that displacement of anomalies in the direction of considerable thickness is available for both optical and electric properties.

Layers obtained on heated substrata have an unstable structure; under the influence of a variety of factors they tend to turn into «normal» layers, i. e. they display the same optical properties as though they were obtained on a non-heated substratum. The conversion sets in upon lengthy exposure of such a layer to air (for days and weeks); very quickly under the influence of iodine vapour and still quicker under the action of vapour of bromine. It should be emphasized here that it is not conversion into halogen silver that is stated to take place; it is true that this phenomenon will indeed take place, but only if halogen vapour should be acting for a very long time. A short-time action of, say, iodine vapour, though it certainly does cause the partial formation of the iodine of silver, yet produces this compound in such a small quantity that the layer still shows the optical properties characteristic of silver (the layer turns from a yellow tint into blue which is characteristic of thick silver layers; iodide of silver is almost colourless in such layers).

Now what explanation can be given to the effect brought about by heating the substratum while the layer is dusted-on? The very method for obtaining

---

* The above-mentioned results were obtained by us in 1936 and in the month of May the same year were reported to the Symposium on Surface Phenomena of the State Institute for Optics. Krautkrämer's Paper was published in 1938 (4).

such layers altogether precludes the possibility of their having an amorphous structure ([5]). In this case one can speak either about a colloidal structure, or about silver entering into chemical combination. The latter surmise is, however, hardly probable since the compounds of silver possible under such conditions as obtained here, will have in respect of their physical properties nothing in common with the films of silver investigated. Consequently, the only conclusion left is that the layers obtained on heated substrata possess a colloidal structure. This supposition finds support in the fact that such layers will scatter light appreciably. As, however, these layers are in their physical properties perfectly similar to the thinnest anomalous layers obtained at room temperature, we must necessarily ascribe a colloidal structure to the latter layers, too*.

We are led to the very same conclusion regarding the colloidal structure of the thinnest layers of silver if we make a comparison between the nature

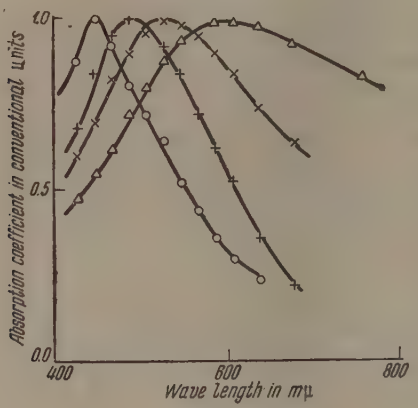


Fig. 1.

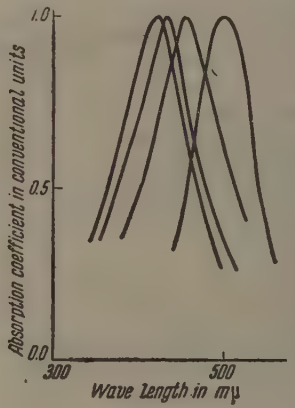


Fig. 2.

of light absorption of these films and that of admittedly colloidal silver within the solid phase. In Fig. 2 are to be found curves of absorption of colloidal silver in KCl (*1* and *2*), glass (*3*) and also silver films of a thickness of 8 mμ (*4*), obtained on heated KCl.

From this figure it may be seen that: a) Absorption maxima of colloidal silver in KCl and glass lie within the region of 420—500 mμ; the absorption maximum for silver in one and the same medium will, with growth in particle size, become displaced into the long wave length region of the spectrum. b) Absorption curves for silver layers lie in the very same region and with growth in layer thickness they, too, give a displacement of the absorption maximum running in the direction of long wave lengths. Thus, these systems exhibit perfect analogy in respect of optical properties and therefore it is but natural to ascribe them an identical structure.

The experimental curves given for colloidal silver in the solid phase coincide sufficiently well in the position of the absorption maximum and the character of its displacement upon increase in size of particles with the curves calculated for these systems on the basis of Rayleigh-Mi's theory. Hence it follows that: 1) even the thinnest layers of silver consist of particles each of which possesses the lattice, as well as the optic constants of a massive metal; 2) when calculating the optical properties of the system: colloidal

* This conclusion, which agrees well with most electronographic observations cannot, of course, be extended to layers obtained at low temperatures of the substrata.

silver at the interface solid—gas, it will be necessary in choosing the respective medium to take the index of refraction of the substratum and not that of air (i. e., to assume that the colloidal particles of silver lie inside the substratum).

The latter assumption is to a certain extent true also from a physical viewpoint. Thus, if silver globules suspended on a tungsten wire be brought into contact with the substratum (glass) and if they are then heated up to a point at which the evaporation of silver starts, then it will be found that, depending upon conditions of heating, it is possible to obtain systems allied in respect of their optical properties, but with a different degree of penetration of silver into glass. The behaviour of layers obtained on heated substrata can also be best of all accounted for on the basis of such an assumption.

Measurements performed by us on the thinnest films of gold have shown that: 1) golden layers also posses a colloidal structure and 2) their optical properties can be approximately described with the help of Rayleigh-Mi's theory.

The structure of the thinnest layers of silver and gold obtained in vacuum, at room temperature of the substratum also appears to be unstable. Namely, such layers upon being exposed to air even for a short time display a rather sharp change in optical properties. Layers of silver are obtainable which seem to be practically colourless in vacuum (we did not succeed in measuring the absorption in the visible region). Upon being brought in contact with the atmosphere, these layers become visibly coloured (yellow in passing light). Measurements effected with thicker layers show that a displacement of the absorption maximum into the long wave length region of the spectrum takes place here. Such a change corresponds to an increase in the size of the particles; we are inclined to explain the recrystallization of the layer by a thermal effect brought about by the heat of adsorption of atmospheric elements, which process is certain to take place here.

The question as to the nature of the metal in these thinnest layers (layers colourless before transformation), i. e. whether there existed here too colloidal particles, but of a smaller size, or whether we are confronted here with atomic distribution of metal, or, finally whether we might not have here a certain intermediate state, will form the subject of another work conducted in connexion with analogous processes in crystals.

Thanks are due to Prof. T. P. Kravetz and M. V. Savostianova for valuable suggestion and guidance in work.

State Institute for Optics.
Leningrad.

Received
7. V. 1939.

REFERENCES

[1] А. Т. Ащеулов, УФН, XX, 481 (1938). [2] M. V. Sawostianowa, ZS. f. Phys., **64**, 266 (1930). [3] T. Fukuroi. Honda anniversary volume, 80 (1936). [4] J. Krautkrämer, Ann . d. Phys., **32**, 537 (1938). [5] J. Kramer, ZS. f. Phys. **106**, 675, 682 (1937).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIV, № 2

PHYSICS

NATURE OF THE LOW-FREQUENCY RAMAN SPECTRUM AND THE SPECTRUM OF THE HEXAMETHYLBENZENE CRYSTAL

By E. F. GROSS and A. I. RASKIN

(Communicated by D. S. Rojdestvensky, Member of the Academy, 2. IV. 1939)

In 1935, Gross and Vuks(1) discovered a new type of Raman spectrum peculiar to some organic substances, the low-frequency Raman spectrum. The Raman lines of that spectrum display two characteristic features:

1) In the new spectrum, the region of the Raman line is adjacent to the Rayleigh line, so that the natural frequencies of the substances they correspond to are low.

2) In the crystal spectrum the lines are quite distinct. Instead of the sharp crystal lines, the spectrum of the same substance in the liquid state displays in the same region a continuous spectrum which has long been known as the Rayleigh line «wings».

On the basis of their experiments, Gross and Vuks suggested an explanation for the new crystal lines and put forward a new hypothesis to account for the origin of the Rayleigh line «wings». They assumed that the new Raman lines were due to the intermolecular oscillations of the crystal lattice, while the Rayleigh line «wings» were produced by similar oscillations in the «remains» of the crystalline lattice (cybotactic groups), to be found, however distorted, in liquid. In assuming the nature of the new Raman lines to be the same, Gross and Vuks regarded the Rayleigh line «wings» as one more proof of the quasi-crystalline structure of liquids. Similar inferences were drawn by Magat(2) in his paper on the Raman spectrum of water. This point of view is supported by Venkateswaran(3) in his paper on low-frequency Raman spectra of sulphur and phosphorus. A number of papers recently published by Sircar and his co-workers(4) confirm in the main the experimental results obtained by Gross and Vuks. The above authors, however, suggest a somewhat different interpretation of their own results and of those obtained by Gross and Vuks. Without denying the fact that the new lines in the low-frequency Raman spectrum are due to intermolecular oscillations, the above writers conclude from their own experiments that those lines have nothing to do with the oscillations of the crystalline lattice, but are produced by the intermolecular oscillations of hypothetical group of «polymerized» molecules which are supposed to be present in the crystalline lattice.

While we do not intend to discuss here in detail neither the experimental results obtained by Sircar and co-workers nor their interpretation (however open to criticism they might be), we describe in this paper an experiment which would seem to us to confirm the hypothesis put forward by Gross and Vuks and to be at variance with the interpretation suggested by Sircar.

A certain confirmation of the hypothesis of Gross and Vuks may be seen in Vuks'([5]) papers on low-frequency Raman spectra of a mixed crystal of para-dichlorbenzene and para-dibrombenzene and of the crystals of two modifications of para-dichlorbenzene. It may also be seen in the experiments of Gross and Komarov([6]) with the vapour of carbon disulphide. The results of the experiments in question point distinctly to the intermolecular nature of the low-frequency Raman spectrum. It would seem to us, however, that the results obtained by Vuks might also be explained from the point of view suggested by Sircar, provided it is assumed that the transition from one modification to another and the formation of mixed crystals is due to some changes within the «polymerized» groups.

In attempting to solve the problem of the nature of the Raman line in the low-frequency spectrum we deemed it essential to perform an experiment with a substance, the crystalline lattice of which would be free from suspicion as to the presence of any «polymerized» groups. In other words, all the molecules that make up the crystal of such a substance should be present in the lattice in absolutely identical conditions and there would be no reason to associate the molecules in any groups. A substance whose lattice has but one molecule for its element can be said to answer these requirements.

Only few organic substances answer these requirements. The most remarkable among these is hexamethylbenzene $C_6(CH_3)_6$ ([7]). The structure of its crystalline lattice has been investigated with X-rays by Lonsdale-Jardley([7]). According to Lonsdale-Jardley's data, the element of the hexamethylbenzene lattice consists of but one molecule. This result is confirmed by crystal chemistry, the inference as to the structure of hexamethylbenzene being based on quite different considerations([8]).

We therefore deemed it essential to submit to investigation hexamethylbenzene, inasmuch as two different methods yield identical results as to the number of molecules in the element of its lattice. A monocrystal of that substance was grown according to the method described in a previous paper of Gross and Vuks. The growing of the crystal had been attended by some difficulties owing to the high melting point of hexamethylbenzene (164°). It was equally difficult to obtain a satisfactory surface of the crystal owing to the high volatility of hexamethylbenzene. The spectra were photographed with a spectrograph having a dispersion of 9 Å/mm in the region of 4047 Å. The photographing revealed a strong fluorescence of hexamethylbenzene. Nevertheless, a photograph of the low-frequency Raman spectrum was obtained, two bands at the frequencies 53 cm^{-1} and 95 cm^{-1} being found in it.

There may be some faint lines besides these two bands, but the continuous fluorescence background in the spectrum of the hexamethylbenzene makes the observation of these faint lines very difficult*.

The fact that hexamethylbenzene crystals give rise to a low-frequency Raman spectrum shows, as this may be seen from the above, that in this case the latter is due to oscillations of the crystalline lattice. There is no reason to suppose that in any other case or in any other substance the low-frequency Raman spectrum may in the main possess a different nature.

Our experiments leave, of course, aside the problem of how «polymerization» may be reflected in the Raman spectrum generally in the case of substances where that phenomenon has been proved to exist by other methods. It is not impossible that in some cases those Raman lines which correspond to oscillations of associated molecules with respect to each other may also

---

* The ultra-violet radiation of the mercury lamp was absorbed by a suitable filter (10% solution of $NaNo_2$ in water). Yet the fluorescence of the hexamethylbenzene was very prominent.

be found in the region of the low-frequency Raman spectrum. Such oscillations are also intermolecular ones, and it would thus be but natural to expect them to appear in a spectral region not far removed from the Rayleigh line. This question, however, calls for special investigation.

The hexamethylbenzene used in these experiments was kindly put at our disposal by Dr. D. V. Tischchenko. We wish to express our thanks to Dr. D. V. Tischchenko for this valuable help in our work.

Physical Institute.
Leningrad State University.

Received
13. V. 1939.

REFERENCES

[1] E. Gross et M. Vuks, Journal de Phys. et le Radium, **7**, 113 (1936); Nature, **135**, 100; 431 (1935). [2] M. Magat, Trans. Faraday Soc., **33**, 114 (1937). [3] C. S. Venkateswaran, Proc. Ind. Acad. (A), **4**, 414 (1936). [4] S. C. Sircar and coll., Ind. Journ. of Physics (1936, 1937, 1938). [5] M. Vuks, Acta Physicochimica URSS, **6**, 11, 327 (1937). [6] E. Gross a. E. Komarov, ibid., **6**, 637 (1937), [7] K. Lonsdale-Jardley, Proc. Roy. Soc., **123**, 494 (1929). [8] O. Hassel, Kristallchemie (1934).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIV, № 2

PHYSICS

ENERGY OF CONVERSION ELECTRONS ARISING IN THE TRANSFORMATION OF BROMINE ISOMERS

By L. I. ROUSSINOW and A. A. YUSEPHOVICH

(Communicated by A. F. Joffe, Member of the Academy, 19. V. 1939)

It has been established by previous research(1) that the isomeric Br-nucleus with a half-period of 4.5 hours emits a soft electron radiation.

The investigation of the characteristic X-rays due to the radioactive transformation of the bromine has shown that the above soft radiation must be put down to internal conversion electrons(2), such as arise during the transition of the metastable isomeric Br-nucleus to the ground state(3).

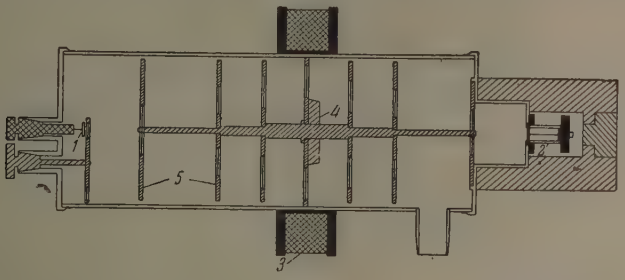


Fig. 1. 1, Source of electrons; 2, Geiger-Müller counter; 3, focusing coil; 4, lead filter; 5, aluminium diaphragms; 6, connexion to the pump.

In order to investigate the tranformation of the isomeric Br-nuclei, the energy of the conversion electrons was determined by means of a magnetic spectrograph with longitudinal focusing.

The focal length of the spectrograph was 25 cm (Fig. 1), a short magnetic coil playing the rôle of the electron lens. The apparatus was evacuated to 10^{-6} mm Hg by means of a diffusion oil pump.

A system of aluminium diaphragms was used to eliminate the stray electrons. A lead filter of 2 cm thickness was placed in the center of the apparatus to protect the Geiger-Müller counter from the direct beam of fast electrons. The homogeneous electron beam selected by the spectrograph was directed along the axis of the counter through a window covered by a cellophane film 1.8 μ thick. The pressure in the counter was about 8 cm Hg. From the electrono-optical characteristics of the apparatus and the dimensions of the target and the windows of the counter it was possible to determine

with an accuracy of 6% the energy of the electrons within an interval of several tens of ekV.

The radioactive source was obtained by irradiating ethyl bromide with slow neutrons, using the cyclotron of the State Radium Institute of the Academy of Sciences of the USSR. The radioactive bromine was extracted by chemical methods and the radioactive deposit was spread over a sheet of mica, 5 μ thick.

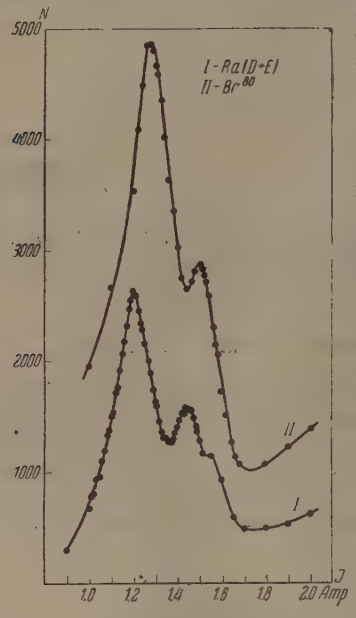


Fig. 2.

The experimental results are represented in Fig. 2. The number of counts during 10 min. (disintegration of Br being taken into account) was plotted against the current strength in the focusing coil. The curve shows that in the bromine, in addition to the electron conversion on the K-level, a conversion on the L-level takes place.

The energies of the conversion on the K- and the L-level have been found equal to 33.5 ± 2 ekV and 46.5 ± 3 ekV respectively. These values are in good agreement with those reported by Valley and McCreary[4] which have come to our notice only after the present research had already been completed.

Within the limits of experimental error, the difference between the energies of the K- and the L-level of the Br-atom determined in the above way coincides with that deduced from the study of X-rays. The spectrograph was calibrated in the soft region of the spectrum, using the data on the energy of the conversion electrons of Ra $(D+E)$[5]. These measurements are also shown in Fig. 2.

The results of the present research confirm once more that the soft radiation of the isomeric Br-nuclei is connected with conversion processes. The fact that conversion takes also place on the L-level makes it possible to see how far the existing theories of nuclear isomerism are in concordance with experimental evidence.

We intend to continue our investigation of the conversion spectra of isomeric nuclei.

Thanks are due to prof. Kourtchatow for his much valuable advice.

Physico-Technical Institute.
Leningrad.

Received
23. V. 1939.

REFERENCES

[1] Roussinowa. Yusephovich, C. R. Acad. Sci. URSS, XX, 647 (1938). [2] Roussinowa. Yusephovich, ibid., XXII, 580 (1939). [3] Hebb a. Uhlenbeck, Physica, 5, 605 (1938), [4] Valley a. McCreary, Phys. Rev., 55, 666 (1939). [5] Alichanian, Alichanov a. Dželepow, Sow. Phys., II, 204 (1937).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIV, № 2

PHYSICS

SOME CONTRIBUTION TO QUANTUM ELECTRODYNAMICS. III

By V. L. GINSBURG

(Communicated by V. A. Fock, Member of the Academy, 29. IV. 1939)

The consecutive application of quantum electrodynamics to problems of the radiation theory leads, generally speaking, to results that seem to differ from those obtained on the ground of the conformity principle.

Let us examine, for instance, the emission of electromagnetic energy by an electron. The range of the probability of this process in the first approximation of Dirac's perturbation theory is equal to:

$$a_{n\lambda} = \frac{\left(ko\left|-\frac{e}{em}\vec{p}\vec{A}\right|n\lambda\right)\{e^{-\frac{i}{\hbar}(\varepsilon_k-\varepsilon_n-\hbar\nu_\lambda)t}-1\}}{\varepsilon_k-\varepsilon_n-\hbar\nu_\lambda} \tag{1}$$

where ε^i is the energy of the electron in the i state, and ν_λ is the frequency of the emitted radiation. Hence, if we neglect the reaction of emission, i. e. if we assume the impulse of radiation as being equal to zero, we obtain, upon transition of the electron from state k to state n, the following expression for emitted energy:

$$U = \sum_\lambda |a_{n\lambda}|^2 \hbar\nu_\lambda = 8\pi e^2\hbar^2\left(\frac{\varepsilon_k-\varepsilon_n}{\hbar}\right)\times$$

$$\times\sum_\lambda \frac{|(\vec{e}_\lambda\vec{x}_{kn})|^2\left\{1-\cos\left(\frac{\varepsilon_k-\varepsilon_n-\hbar\nu_\lambda}{\hbar}\right)t\right\}}{(\varepsilon_k-\varepsilon_n-\hbar\nu_\lambda)^2}. \tag{2}$$

In the case of a uniformfly moving electron with impulse $\vec{p}_0$, neglecting reaction $\varepsilon_k-\varepsilon_n=0$ and using (1), we obtain:

$$U_p = \frac{8\pi e^2}{m^2}\sum_\lambda \frac{(\vec{p}_0\vec{e}_\lambda)^2}{\nu_\lambda^2}\{1-\cos\nu_\lambda t\}. \tag{3}$$

Expression (3) coinciding with the one obtained in a classical way for a uniformly moving electron (I, 12) and expression (2) will, upon effecting summation over all frequencies, be found to diverge, i. e. infinitely great energy will be emitted*. The reason for this circumstance was already made clear in communication I(1) and lies in the character of the initial conditions made use of in quantum electrodynamics, according to which at moment

* With calculations by the conformity method we obtain that a uniformly moving electron fails altogether to emit, while a fixed electron is liable to emit an amount of energy which is equal to that obtained from expression (2), upon summation being effected only over the region of resonance.

$t=0$ a moving electron is available which fails to be surrounded by the transverse electromagnetic field it entrains. In this case the energy of field (3) or the energy obtained from (2) upon integration outside the resonance region bears a character altogether different from the energy of emission obtained from (2) upon integration over the resonance region. This latter part of energy of field is liable to grow in proportion to time and is equal to:

$$U_{st} = (\varepsilon_k - \varepsilon_n) A_n^k t, \qquad (4)$$

where A_n^k is the probability of spontaneous emission. The dependence upon time of expression (3) as well as of part of expression (2) taken outside the resonance region is totally different. Thus, if the spectrum be broken off at a certain frequency ν_{max}, U_p becomes equal to expression (I, 12′); in this case $\left(\frac{dU_p}{dt}\right)_{t=0} = 0$, whereas $\frac{dU_s t}{dt} = (\varepsilon_k - \varepsilon_n) A_n^k$. Further, when $t \to \infty$ U_p tends to a definite limit, while U_{st} increases continuously. The above character of the dependence of U_p (and also $U - U_{st}$) upon time is perfectly obvious. U_p is the energy emitted by the electron assuming that the entrained field is absent when $t=0$, i. e. at the outset of the process.

Should the interaction be included adiabatically, the electron would emit (see I) only its own entrained field. In the theory of emission, on whose basis expression (1)—(3) are obtained, use is, however, made of instantaneous inclusion of an interaction equivalent to an instantaneous imparting of the terminal state of motion to the electron. In course of this, apart from the entrained field, a light field possessing a continuous spectrum is also emitted. The light field whose appearance is due to the inclusion of interaction will, just as well as the entrained field, tend with $t \to \infty$ to a certain stationary state, which fact actually takes place.

Hence it seems advisable to introduce the notation of «true» light emission referring to the essentially non-stationary part of the emission.

Only this part of energy lies outside any connexion with the initial conditions of the problem, and this energy being obtained from the full expression for emission energy by segregation of the resonance region is exactly equal to the emission energy found on the basis of the conformity principle. It should be emphasized that a thoroughly analogous situation is available in the classical theory. Let us by way of illustration examine the emission of an electron mowing at constant velocity $\vec{v}$ within a medium with a refraction index $n(\nu) = \sqrt{\varepsilon(\nu)}$. From wave equation

$$\Delta \vec{A} - \frac{n^2}{c^2} \ddot{\vec{A}} = -\frac{U_-}{c} n_e \delta(\vec{x} - \vec{x}(r)),$$

where $\vec{x}(r)$ is the radius-vector of the electron, expanding $\vec{A}^{tr}$ into a series (I, 4), where only c is to be substituted with $\frac{c}{n}$, we obtain an equation for the co-ordinates of the field:

$$\ddot{q}_{\lambda_i} + \nu_\lambda^2 q_{\lambda_i} = \frac{\sqrt{8\pi e}}{n} (\vec{v}\vec{e}_\lambda) \{\delta_{1i} \cos \vec{k}_\lambda \vec{x}(r) + \delta_{2i} \sin \vec{k}_\lambda \vec{x}(r)\} \qquad (5)$$

where $\nu_\lambda = \frac{|\vec{k}_\lambda| c}{n}$ and in the case of a uniformly moving electron $\vec{x}(r) = \vec{v} t$. If $n=1$, the resonance in (5) will be impossible, since $\vec{k}_\lambda \vec{v} = \frac{\nu_\lambda}{c} v \cos(\vec{k}_\lambda \wedge \vec{v}) < \nu_\lambda$.

In this case (5) admits of a stationary solution, i. e. a solution which describes motion with invariable energy of field, while under definite

initial conditions we obtain an expression of the type of (3)*. In this case there is no «true» emission whatever. An altogether different situation will be possible if $n>1$. In this case, resonance with sufficiently high value of $v(v<c)$ is quite possible, its conditions being evidently as follows:

$$v\cos(k\hat{\vec{v}}) = \frac{c}{n}. \qquad (6)$$

The solution of (5) for this case involves solutions which are non-stationary in the region of frequencies near resonance. The energy of the field within this region grows in accordance with (4) and electron emission takes place.

The case above referred to corresponds to that recently discovered by Cherenkov and explained by I. E. Tamm and I. M. Frank([2]) which consists in the emission of light by fast electrons moving within a given medium with constant velocity. Formula (6) coincides with the conditions of emission obtained by the above authors, and their expression (15) for the energy emitted along path l can be readily obtained from the integration of system (5) and a subsequent calculation of the energy of the field.

Thus, we see that the condition for the existence of resonance** is a condition for «true» emission, both in classical and in quantum theories. Considerations based on the principle of conformity in the form in which they are applied in the emission theory refer, by virtue of their very significance, to «true» emission only. Therefore, we are bound to recognize the results of the quantum theory of emission as being in full agreement with this principle, while the limitations of the integration region by the resonance region should, when calculating the emitted energy, be regarded not as a kind of prescription, but as an operation dictated by the very essence of the problem.

In connexion with the discussion of the problems of the emission theory let us touch upon the question of the nature of spontaneous emission. It is highly important to make it clear whether spontaneous emission is a quantum effect or whether it is, quite on the contrary, a classical one. We do not know of this question having been discussed by any other author, except by Weisskopf([3]). Weisskopf maintains that spontaneous emission is «the direct consequence of the existence of zero states of field oscillators», and that «a spontaneous emission of light is a photon emission brought about by zero oscillations of empty space». Thus, inasmuch as zero oscillations are a purely quantum effect, it is legitimate to think that spontaneous emission may also be held to be such by Weisskopf. More than that, he ventures to consider spontaneous emission as induced by a zero field, endeavouring in this way to connect the ratio of Einstein's coefficients A_n^k and B_n^k with the density of zero energy. However, as it follows from the subsequent, there is no reason for considering spontaneous emission as being a quantum effect. According to quantum theory, spontaneous emission makes its appearance under the following conditions: at moment $t=0$ a certain electrodynamic system is considered in a stationary state w i t h a b s e n c e o f i n t e r a c t i o n w i t h f i e l d, the energy of the field being at this moment assumed as equal to zero. Up from moment $t=0$ the development of the system is investigated with already i n c l u d e d i n t e r a c t i o n, whereupon it is found that when $t=0$ the energy of the field has become

* Expression (3) will be exactly obtained in the classical theory, provided Hamilton's form of equations is resorted to and on the condition that certain simplifying assumptions are made (see I).

** With spontaneous motion of the electron the availability of resonance, as can be easily seen from (5), is likewise a condition for the existence of solutions growing with time, i. e. of true emission.

different from zero. It is this very emergent energy which is held to be radiated by spontaneous emission. It may be readily seen that with the same statement of the problem the same result will also be arrived at on the basis of the classical theory. In fact, postulating for the solution of system (5) such initial conditions under which, with $t=0$, the energy of the field is equal to zero, we shall obtain that this energy has already made its appearance when $t>0$. The point is simply that the non-stationary state of a full electrodynamic problem is taken as the initial one, which state can certainly be a stationary state in a mechanical problem. If there is the possibility of resonance, then stationary initial conditions will be absent and the system will emit (within the resonance region of frequencies) in accordance with law (4) without any dependence at all on this or other selection of the initial conditions. It is for this very reason that the initial conditions postulated in quantum electrodynamics (see I) which are definitely known to be non-stationary, prove to hold good for the theory of emission. We thus see that spontaneous emission is the result of a specific statement of the problem and may be available in any theory.

Such a point of view does not preclude the possibility of connecting—within the limits of every theory—spontaneous emission with this or other of its elements. In particular, attempts within the quantum theory can be made to establish a relationship between spontaneous emission and zero fluctuations. Formally such a relationship subsists, because the probability of spontaneous emission is determinable by the matrix element of transition from zero state to the first one, i. e. depends upon Ψ_0 (the function of zero state). So far this circumstance fails to provide us with a possibility to pass any judgment on the essence of the problem, since it is clear that Ψ_0 occurs, for instance, also in the expression for the probability of the absorption of a single photon in the absence of other photons in the field and accordingly the availability of Ψ_0 in the expression of a theory has of itself a merely formal significance. The above-mentioned attempt of Weisskopf to consider spontaneous emission as one called forth by the energy of a zero field, bears likewise a purely formal character and is moreover on the whole not clear to us, since, on the one hand, the density of zero energy is but one half of that necessary for obtaining a correct ratio of coefficients A_n^k and B_n^k , and on the other hand—which is by far more important—since zero energy is infinite and thus should and can be excluded from the Hamiltonian operator without any consequences whatsoever.

Let us, however, remark in connexion with this that the exclusion of zero energy from the Hamiltonian operator of quantum electrodynamics by no means signifies doing away with all difficulties associated with the zero state of the oscillators of the field. The availability of these states involves a divergence between the highest approximations of the energy of interaction of field and electron, and it is hardly possible to do away with these divergences, even so much as formally [see e. g. communication II(4)]. This leaves us with the belief that it is a clear comprehension of the true part taken by zero oscillations, this fundamental and basic consequence of quantization, that will lead to further progress of theory, while the attempts to do away with these oscillations seem to be entirely hopeless.

Scientific Research Institute of Physics.
Moscow State University.

Received
25. V. 1939.

REFERENCES

[1] V. L. Ginsburg, C. R. Acad. Sci. URSS, XXIII, No. 8 (1939). [2] I. E. Tamm a. I. M. Frank, ibid., XIV, 107 (1937). [3] V. Weisskopf, Naturwissensch., 23, 631 (1935). [4] V. L. Ginsburg, C. R. Acad. Sci. URSS, XXIII, No. 9 (1939).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIV, № 2

PHYSICS

ON «DRY FRICTION» FORCES

By S. E. HAYKIN, L. P. LISSOVSKY and A. E. SOLOMONOVICH

(Communicated by L. I. Mandelstam, Member of the Academy, 26. V. 1939)

The forces of «dry friction», i. e. the forces arising between the surfaces of solids in contact, are manifest both in the case when contiguous surfaces slide with respect to one another (so-called «sliding friction» forces) and in the case when sliding between the surfaces in contact has not yet set in («quiescent friction» forces). A clue to the understanding of the origin of «sliding friction» forces lies undoubtedly in a comprehension of the nature of «quiescent friction» forces and in a study of the mechanism underlying the transition of «quiescent friction» forces into «sliding friction» forces. Existing theories of dry friction ([1–4]) contain attempts to link these two types of forces. In fact all these authors introduce elementary forces of interaction between contiguous surfaces, assuming these forces to bear a purely positional character. Regarding the character of the elementary forces (the dependence of these forces upon displacement), various surmises are set forth, however, all these presumptions may in the last account be reduced to an explanation of the basic features of quiescent friction and sliding friction (in particular, the non-conservative character of the force of «sliding friction»). However, all these theoretical considerations remain as yet purely hypothetical, this being due to the almost total absence of experimental investigations into the nature and the character of the forces underlying quiescent friction and the mechanism governing the transition of these forces into sliding friction. We are aware of only two works devoted to this question, namely those of Rankin ([5]) and Tomlinson*. While making use of different methods, both these authors disclose positional (and consequently conservative) forces in the case of small displacements of conterminal surfaces. However, Rankin suceeded in detecting these conservative forces in displacements of up to $8 \cdot 10^{-6}$ cm, while Tomlinson arrived at the conclusion that the forces of interaction between displaced surfaces already cease being conservative in the case of displacements of less than $1 \cdot 10^{-7}$ cm. Thus, the question of the transition of «quiescent friction» into «sliding friction» (positional forces passing into non-conservative forces) remains so far unsettled. The difficulty of investigating this problem lies in the fact that transition

* The recently published works by Bowden and Leben ([6]) on the nature of friction only corroborate the ere long established fact that quiescent friction cannot exceed a certain definite magnitude and that sliding friction is in most cases less than this maximum quiescent friction. No new suggestions relating to the transition from quiescent to sliding friction are to be found in this paper.

begins at such small displacements which defy any direct observations under the microscope. On the other hand, a study of interaction forces between contiguous surface with small static displacements or with slow motion is greatly handicapped by the fact that, along with displacements between the surfaces of contact, elastic deformations may also arise inside the very contiguous surfaces. The elastic forces apt to occur inside bodies in contact may easily disguise those very forces of interaction between two surfaces which are to be investigated.

L. I. Mandelstam and one of the authors of the present paper (S. E. Haykin) proposed rapid mechanical oscillations for investigation of frictional forces with small displacements. One of the advantages of this method consists in the fact that with high frequencies (in so far as quite appreciable velocity ranges and considerable acceleration ranges correspond to very small displacements) the second body in contact with the given oscillating body has not to be fastened, which excludes the danger of elastic displacements within the very bodies along with displacements of the contiguous surfaces (for details see footnote, page 8). Oscillations of a piezo-quartz resonator appear to be the most suitable for carrying this method into practice. Owing to the piezo-effect it is very convenient to produce rapid mechanical oscillations in them. Placing a solid body on the surface of the oscillating piezo-quartz resonator, we introduce new forces of interaction between the quartz and the superimposed body which must necessarily, in this or other way, alter the oscillatory properties of the piezo-quartz resonator. Since piezo-quartz resonators possess a very sharp curve of resonance, the shape of this curve appears to be highly sensitive even to the slightest disturbances introduced by the forces of interaction between the quartz and the superimposed body. Changes in the shape of the resonance curve afford the possibility to judge of the character of the interaction forces between the quartz and the superimposed solid body, even with very slight displacements.

This method proved indeed to be most simple and convenient. In fact, it enabled us to have some insight into that very region of displacements where the transformation from conservative to non-conservative forces is liable to take place and though this region has not yet been fully investigated by us, yet the obtained results warrant certain new conclusions as to the character of the forces brought into play between two contiguous surfaces with very small displacements and, moreover, to outline the general picture of transition from «quiescent friction» to «sliding friction»*.

The suggested procedure was as follows: in order to obtain oscillations in the piezo-quartz resonators a modulator valve generator was made use of, one of whose side frequencies could be made to coincide with the own frequency of the piezo-quartz resonator. Such an arrangement warranted, on the one hand, the stabilization of the generator and in this way also permitted of securing the necessary stability of frequencies acting upon the quartz; while, on the other hand, it excluded the possibility of there being set up a reverse effect of the quartz affecting the generator. The resonance curves of the piezo-quartz resonator both without any charge and

---

* Since the surfaces used in our experiments were not preliminarily subjected to any treatment or cleaning, it is possible to «speak here about the contact of solid bodies» only in a conventional manner; under the conditions of our experiments actual contact, in all probability, took place, only over a small part of the whole surfaces, various surface phenomena (impurities on surface, air-layer etc.) possibly also playing a significant part.

Accordingly, our experiments do not permit of drawing any conclusions as to the phenomena liable to occur in the case of well cleaned surfaces being in faying contact; yet our experiments are of interest just because they refer to a case which is often met with under practical conditions.

with charges of different size and material, were regarded by means of smooth variation of the modulation frequency. The charges were made of steel, aluminium and ternary brass and had the shape of parallelepipeds, the faces of which were several millimeters long.

With the help of an additional load it was possible to alter the pressure of the superimposed charge upon the quartz without altering its dimensions. Additional pressures came usually to several grams, and occasionally to several score grams. The experiments were run with two quartz unites at frequencies of 70 000 and 150 000 Hz.

Evidently, two types of displacement between solid surfaces in contact are possible—tangential displacement which corresponds macroscopically to the sliding of surfaces, and normal displacement which corresponds macroscopically to the two surfaces being rent asunder. Both these possibilities can be materialized with the help of piezo-quartz resonators. In the first case the charge is placed on the side surface of the lengthwise oscillating quartz, while in the second case it is superimposed upon its butt surface*.

Fig. 1.

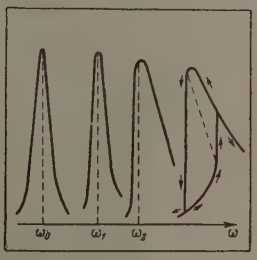


Fig. 2.

Notwithstanding the fact that these two cases would seem to be essentially different, the influence of the charges upon the character of the piezo-quartz resonance curves was found to be almost equal in both cases and this not only qualitatively, but even quantitatively. Accordingly, all the below stated results of experiments refer in an equal measure to both tangential and normal displacements.

The basic results are in brief as follows: in placing upon the quartz charges made of diverse metals and alloys (steel, aluminium, ternary bronze) the resonance frequency of quartz is always enhanced, while the damping of quartz increases but in a very small measure. The resonance curve has a shape which is usual for a linear resonator. If with unchanged pressure upon the charge the amplitude of quartz oscillations is increased, then transition from the normal resonance curve (with small amplitudes) to a non-symmetrical curve is observed, whereupon there appear in the resonance curve non-univocallity and breaks (Fig. 1). On the other hand, with increase in pressure acting upon the charge, the rise of the resonance frequency (different for different metals) becomes continuously more and more noticeable. With further increase, apart from rise in frequency, distortion (break of symmetry) of the resonance curve also becomes manifest, whereupon (with yet greater pressure) there appear in the curve non-univocallity and breaks (Fig. 2).

---

* It was confirmed by special control experiments that quartz actually oscillates along its length and that in the first case we deal practically only with tangential displacements, while in the second case only with normal ones.

From the results obtained the following general conclusions may be drawn.

1. Since the charge raises the resonance frequency of quartz and but very slightly enhances its damping, this means that the forces of interaction between the surfaces of the quartz and the charge are equivalent to an additional elastic fastening, i. e. these forces will in the case of small displacement be almost purely positional (and therefore conservative).

2. Since with small amplitudes of oscillations frequency becomes displaced, while the shape of the resonance curve remains unaltered, this means that the forces of interaction will in the case of small displacements be linear (in conformity with Hook's law). The distortion of the shape of the curve as well as the appearance of breaks with increased amplitudes of oscillations show that with growth of displacement the forces of interaction cease to be linear, and this deviation from linearity is such that these forces show a slower rate of growth than do displacements (the «loop» in the resonance curve lies on the side of low frequencies).

3. Since with fixed amplitudes, increase of pressure raises frequency displacement and leads to the appearance of asymetry, the elasticity of the forces of interaction between the surfaces will consequently grow with increase in pressure and at the same time the non-linearity of these forces will grow, too.

4. Since with normal displacements the same phenomena are observable as with tangential displacements, this means that the forces of interaction between the surfaces being displaced bear in both cases an equal character.

The magnitudes of the relative displacements of surfaces cannot be measured in a direct way. Yet, as simple calculations show, the charges in both cases (for tangential and normal displacements) will remain practically immobile* and, accordingly, the amplitudes of the relative displacements of the surfaces will coincide with the amplitudes of the oscillations of the piezo-quartz. These latter can be determined approximately by indirect methods (from stresses at the side-bands of quartz). In our experiments these amplitudes lie within the range of $1 \cdot 10^{-6}$ to $1 \cdot 10^{-5}$ cm.

Thus, it may be considered as proved that with displacements of the order of $1 \cdot 10^{-6}$ cm (for various classes of bodies this magnitude appears of course, to be somewhat different) the interaction forces between contiguous surfaces bear a purely elastic character—this is indeed the field of quiescent friction. With increase of displacements these forces cease to be linear and liable to grow more slowly than displacements will do. Should in the subsequent the rate of growth of the forces be slowed down in an ever-increasing measure and finally altogether stopped, then, evidently, either sliding (with tangential displacement) or breaking off (with normal displacements) will arise. For this reason the appearance of marked non-linearity in the forces of interaction (namely, a progressive slowing down of the rate of growth of the forces) may apparently be indicative of the approach of sliding. Further increase of the amplitudes of oscillations would necessarily soon entail the appearance of sliding which would become manifest in a sharply marked increase in the damping of the quartz (at least in the case of tangential displacements). Unfortunately, we have not yet succeeded in examining thoroughly, by means of our method, all this region of transi-

* The dimensions of the charges are small as compared with the length of the elastic wave which has to be excited in them at a frequency of 150 000 Hz (and so much the more at 70 000 Hz); wherefore the charges can move only as a whole. But, on the other hand, the mass of the charge for these frequencies is already so considerable in comparison with the elasticity of the interaction forces between the surfaces that the charges remain practically immobile.

tion to sliding (since we met with certain practical difficulties in obtaining great amplitudes in the oscillations of quartz), but experiments concomitantly conducted by another method * confirm the fact that the small displacements to which our distorted resonance curves correspond, lie in that very region of transition from quiescent to sliding friction. This other method just referred to consisted in measuring the limiting angle of friction for the charge lying on oscillating quartz. If between the surfaces and the charge sliding already exists (with tangential displacements), quiescent friction ought to be absent and the limiting angle should be equal to zero. Likewise, with normal displacements, if the surfaces are torn asunder, the limiting angle should be equal to zero as well. Thus, transition to sliding or tearing off should be accompanied by a reduction of the limiting angle of friction. As experiment has shown, it is just at those great amplitudes to which our distorted resonance curves correspond, that the limiting angle begins to diminish rapidly. Consequently, transition from elastic «enmeshments» to sliding (or to tearing off) occurs in one and the same region of displacements, and at the same time a deviation from linearity in the forces of interaction between the surfaces becomes noticeable. This deviation from linearity (a slower growth of forces than of displacements) accounts for the appearance of sliding or tearing off.

The authors hope that their method will in the subsequent afford possibilities for studying in greater detail the character of the forces of quiescent friction and for elucidating the mechanisms governing the transition from quiescent friction to sliding friction.

Institute of Physics.
Moscow State University.
Laboratory of Oscillations.

Received
26. V. 1939.

REFERENCES

[1] Prandtl, Gedankenmodell zur kinetischen Theorie der festen Körper. [2] Frenkel, ZS. f. Ph., 3 (1926). [3] Tomlinson, Philos. Mag., No. 7, 905 (1939). [4] Deriaguin, ZS. f. Ph., 88 (1934). [5] Rankin, Philos. Mag., VIII, 806 (1926). [6] Bowden a. Leben, Proc. Roy. Soc., 169 (1939).

* The experiments were made by Brandt, undergraduate.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIV, № 2

TECHNICAL PHYSICS

DETERMINATION OF MINIMUM WORKING WAVE LENGTHS FROM MEASUREMENTS OF THE IONOSPHERE

By K. M. KOSSIKOV

(Communicated by M. V. Shoulejkin, Member of the Academy, 28. IV. 1939)

The optimum or most advantageous wave length for radio-communication between two points is one which sustains the least amount of losses while travelling through the lower ionized layers, and which is at the same time well reflected from the upper layers. Both theory and experiment show that losses in the lower layers will be the less, the shorter will be the wave length applied. On the other hand, provided it has sufficient length, a wave will be well reflected from the upper layers.

Consequently, the ionized layers—the upper, reflecting one and the lower, absorbing one—will in a certain measure bring about mutually conflicting requirements. The two requirements will be mathematically expressed in the following way:

$$E = \frac{3 \cdot 10^5 \sqrt{W}}{d} e^{-k\lambda d}, \tag{1}$$

$$f = \frac{f_{cr}}{\varphi(\gamma)} \tag{2}$$

where $k = F(N. S)$; N, electronic density of layer; S, number of collisions in layer; d, distance in km; W, power in kW; f, working frequency; f_{cr}, critical frequency with straight incidence upon layer; γ, angle of incidence of ray upon layer (Fig. 1).

From expression (1) it will be seen that the power of reception will be the greater, the shorter the wave length used. Hence it becomes obvious that the optimum wave length will be the shortest of all those which satisfy relationship (2). It is also evident that the optimum wave length depends upon the distance between the two points and will vary continuously following changes in the electronic density and the height of reflecting layer.

The determination of the optimum wave length will therefore be of the highest practical significance. Numerous measurements of the intensity of the fields of stations over different directions conducted by the Scientific Research Institute of the People's Commissariat for Communication show that the damping of waves is better expressed in expression (1), if in the index of the exponential factor wave length λ is taken at a power equal to unity. We fail to know with sufficient certitude how damping is distributed among different layers of the ionosphere, and we therefore on the whole refer absorption to layer E. For the same reason absorption coefficient k, being empi-

rical, expresses the aggregate influence of all the layers. However, for daytime, the magnitude of this coefficient follows satisfactorily density of layer E. If the assumption is made that the height of layer E remains almost the same, then obviously the change in the magnitude of coefficient k will depend precisely upon the density of layer E. Hence no particular difficulties will arise in obtaining the relationship between the absolute magnitude of this coefficient and the altitude of the sun over the horizon.

Inasmuch as expression (1) seems to lay claims in favour of the shortest wave lengths, it will be of the utmost importance to determine the maximum working frequencies. Electronic density and height of maximum density change continuously. The reasons for these changes can, as is known, be listed as follows: a) changes in radiation, b) expansion and contraction of atmosphere due to heating and cooling. The difference in electronic densities and layer heights at the same hours of different days may amount to considerable magnitudes even on comparatively calm days.

Under such conditions the determination of maximum working frequency becomes of particularly high importance, since work on minimum wave lengths possesses quite a number of advantages, such as: 1) with maximum frequency the reception is stronger, which fact follows from expression (1) and is confirmed by numerous measurements; 2) the influence of atmospheric disturbances is less at the highest frequencies; 3) in working with the highest frequencies, disturbances from other stations are considerably less felt, which fact is explained both by the conditions of propagation then available and by the range of higher frequencies being less occupied; 4) in working with the highest frequencies, the influence of echo-signals is reduced, which is important for photo-telegraphic communications as well as in telegraphic work at very high rates of speed.

Measurements of critical frequencies and of effective altitudes of the ionosphere permit to determine with a sufficient degree of accuracy the maximum working frequencies provided the distance and the direction of the communication lines are known. In conformity with the laws of geometrical optics and the laws of the reflexion of waves, this determination may be effected from the expression:

$$f=\frac{f_{cr}}{\cos\gamma} \qquad (3)$$

where f_{cr}, critical frequency at straight incidence of wave upon layer; f, critical frequency at oblique incidence of wave upon layer; γ, angle of incidence of wave upon layer.

However, in reality we deal with a medium having a variable index of reflexion and therefore expression (3) evidently calls for a certain correction.

Indeed, if H is height of maximum electronic density, α—angle of incidence of wave upon lower boundary of reflecting layer (Fig. 2), then the reflexion of the wave may be characterized by angle γ_1 and height H^1. This height is in the case of oblique incidence fictitious and will, therefore, be but little fit for practical application. It will be more convenient to use height H which may be obtained for any time of the day by measuring the critical frequencies.

It is possible to admit that the ray falls upon the ionized layer not at point B, but at point C and is reflected at point D at angle γ. For passing over from angle γ_1 to angle γ we might evidently write out expression (3) in the following form:

$$f=\frac{f_{cr}}{\frac{\cos\gamma_1}{\cos\gamma}\cos\gamma}. \qquad (4)$$

Let us denote ratio $\frac{\cos\gamma_1}{\cos\gamma} = q$. It will be easy to assume that value q will depend upon the gradient of electron density. Calculations show that by taking different thickness Z (Fig. 3)

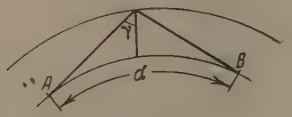


Fig. 1.

Fig. 3.

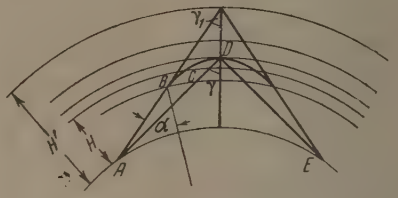


Fig. 2.

of the ionized reflecting layer F, with a parabolic form of its layer boundary, we obtain $q=1.25—1.45$.

Assuming $q=1.35$ (the mean value between them) we obtain rather satisfactory coincidence with the true conditions of travel of radio-waves. Evidently for layer E this coefficient should be less. It approximately equals 1.1. Expression

$$f_p = \frac{f_{cr}}{\cos\gamma q} \qquad (4')$$

may be represented in the form of graphs. Knowing from graphs constructed according to expression (4′) crirtical frequency and height of maximum electronic density of the ionized reflecting layer, we determine directly maximum frequency which is reflected from layers F, F_2, or from layer E. The convenience of the use of this method consists in the fact that the computation graphs are constructed once and for all, and are frequently serviceable during disturbances of the ionosphere, when critical frequencies and heights of reflexions are apt to undergo changes. Such graphs can also be very well used in the case of the reflexion of waves at points with considerable difference in latitude.

The application of such graphs in radio-communications controlled by the Moscow Wireless Centre has been put into practice since the beginning of 1938 (the working out of the method was started in the autumn of 1937). The results of the computation according to the given formula give a divergence from the graphs published in Proc. I. R. E. within the limits of 5—10%.

Calculations on the determination of coefficient q were performed with the participation of the co-workers of the Distribution Laboratory of the Scientific Research Institute of the People's Commissariat for Communication, technicians N. N. Savelieva and S. I. Goldfeld and engineer V. M. Firsov who also took part in drawing comparisons of calculated values with the actual data for the travel of radio-waves. The practical application of the graphs was carried out by shift engineers of the Moscow Radio Centre Bulgakov, Khotianovich, Dmitrieva and others under the general supervision of the Chief of the Moscow Radio Centre, engineer L. M. Kariakin.

The materials of the measurements were obtained from the Tomsk Ionosphere Radio-Station.

Received
20. V. 1939.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIV, № 2

GEOPHYSICS

A MORE PRECISE DETERMINATION OF HUMIDITY FROM THE RECORDS OF AEROLOGICAL SOUNDS

By V. G. DYBCHENKO

(Communicated by O. J. Schmidt, Member of the Academy, 11. V. 1929)

In his work «A more precise determination of humidity from the records of aerological sounds» Prof. W . W. Schoulejkin has suggested and theoretically founded a method by whose means it is possible to decipher hygrograms received during the ascensions of aerological sounds. As shown by investigations, the hygrometer requires about 4—5 min. to record the surrounding humidity. It is natural that a sounding baloon ascending with the velocity of 10 m/sec. takes shorter intervals of time to cross strata with varying humidity; consequently, the readings of the hygrometer will differ from reality.

Prof. Shoulejkin's method enables us to calculate the real reading of the hygrometer S (which corresponds to the surrounding humidity) from a given reading of the hygrometer A, at a given time t.

$$S = A + \frac{1}{k}\frac{dA}{dt} \qquad (1)$$

in case

$$A = S + \Delta l^{-kt} \qquad (2)$$

Sign + corresponds to the sorption of air moisture by the hair, sign — to drying.

The aim of our work consists in:

1) verifying whether in case of sharp humidity variations the readings of a hygrometer really follow the law (2);

2) finding the coefficient k in formula (1).

For this purpose we have carried out the following experiment.

The hygrometer was placed into a special case shaped like a Chinese lantern. By lifting or lowering the latter we could either isolate the hygrometer from the surrounding air, or place into it. The case containing the hygrometer was placed into a hermetically closed Bonguards' apparatus. Lifting and lowering of the case was brought about by means of a special mechanism.

When the case was lifted, the position of the needle was fixed according to the humidity of the surrounding air. After that the case was lowered and the surrounding humidity changed for about 20%. Now by rapidly lifting the case the hygrometer was transferred under new conditions; the

humidity changed abruptly. After this the readings of the hygrometer were recorded at short intervals.

Curve *1* in Fig. 1 corresponds to the increase of humidity, curves *2* and *3*, to the drying. The time is plotted along the axis of the abscissae, relative humidity in per cent along the axis of the ordinates.

At the first sight at the curves one may note that the readings of the hygrometer follow the exponential law pointed out by W. W. Shoulejkin. It is noticeable, too, that the absorption curve shows a steeper rise than the drying curves, i. e. the process of absorption is more intense than that of drying. The latter is accounted for by complicated processes of hysteresis in the hair itself, and partly by the fact that in the case of absorption the

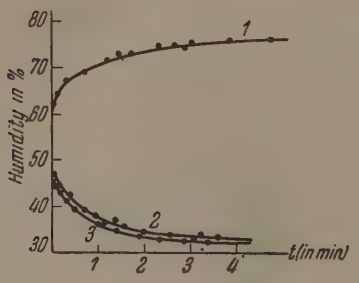


Fig. 1.

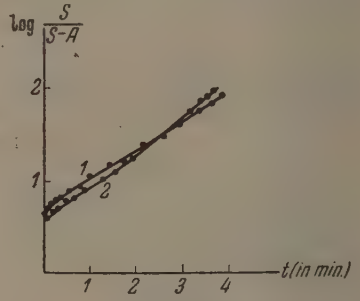


Fig. 2.

needle itself contributes to the lengthening of the hair; it seems to stretch it out, while in the case of drying it resists to the contracting.

Now let us examine our curves. Let us plot t along the axis of the abscissae and $\log \frac{S}{S-A}$ along the axis of the ordinates (Fig. 2).

The lines obtained may be considered as straight lines with a sufficient degree of accuracy. Consequently, equation (2) is corroborated. It should be noted that at the beginning (during the first 5—7 sec.) the readings of the hygrometer rather deviate from the exponential law. At this state the coefficients found by us prove to be somewhat exaggerated. The divergence is the larger, the sharper the leap of humidity. In case of moderate leaps (about 10%) which are the largest observed under natural conditions, this inaccuracy is smoothed.

Thus, 1) the behaviour of the hygrometer under unstationary conditions follows law (2) exposed by W. W. Schoulejkin, consequently, the real humidity S can be determined by the readings A making use of formula (1); 2) for the absorption $k_1 = \frac{1}{80}$ sec.$^{-1}$, for the drying $k = \frac{1}{70}$ sec.$^{-1}$.

Institute of Theoretical Geophysics.
Academy of Sciences of the USSR.

Received
15. V. 1939.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIV, № 2

CHEMISTRY

ON THE OXIDATION OF COMPLEX COMPOUNDS OF PLATINUM WITH HYDROGEN PEROXIDE

By A. V. BABAJEVA

(Communicated by N. S. Kurnakov, Member of the Academy, 19. V. 1939)

In a paper dealing with the oxidation of complex compounds of platinum (1) it was stated that we had succeeded in isolating hydroperoxo-compounds of the type $[(NH_3)_2\,PtCl\,(OOH)_2\,X_2]$. These compounds were obtained by the action of 30% hydrogen peroxide upon non-electrolytes of bivalent platinum.

It is well known that the oxidation both of non-electrolytes and electrolytes with diluted (10%) hydrogen peroxide leads to the formation of dihydroxo-derivatives.

In the present paper the interaction of electrolytes of complex compounds of bivalent platinum with concentrated (30%) hydrogen peroxide has been studied. When carrying out the oxidation under conditions described in the previous paper, we could not discover the formation of hydroperoxo-compounds. Dihydroxo-compounds were found to be in all cases the main oxidation products. But together with them, some other compounds were formed and that indicated a more complicated course of this reaction. Chugajev and Khlopin (2) treated ammonium platinichloride with 10% hydrogen peroxide and obtained a dihydroxo-salt $(NH_4)_2\,[PtCl_4\,(OH)_2]$. The same authors obtained the salt $[(NH_3)_4\,Pt]\,[PtCl_4(OH)_2]$. Greenberg and Filinov (3) synthetized the dihydroxo-salt of silver $Ag_2\,[PtCl_4\,(OH)_2]$, the ethylenediammonium salt $\left[\begin{matrix}CH_2—NH_3\\ |\\ CH_2—NH_3\end{matrix}\right]\,[PtCl_4\,(OH)_2]$, the methylammonium salt $(CH_3NH_3)_2\,[PtCl_4\,(OH)_2]$ and the thallium salt $Tl_2\,[PtCl_4\,(OH)_2]$. Though it seems strange, but they were unable to isolate the potassium salt $K_2\,[PtCl_4\,(OH)_2]$. The authors stated that usually, when the solutions containing the products of the oxidation of potassium platinichloride with hydrogen peroxide were evaporated, it was invariably the potassium platinichloride that separated.

Our purpose was to obtain and to isolate this salt, and to explain the reasons for its unexpected behaviour during the evaporation of the solution. In the present investigation we have been able to establish that $K_2\,[PtCl_4(OH_2]$ could be isolated only if both the corresponding temperature regime and the hydrogen peroxide concentration were controlled.

Oxidation of $K_2\,[PtCl_4]$. Crystals of potassium platinichloride were treated under stirring with an excess of 30% hydrogen peroxide. The reaction vessel was cooled with snow. Under these conditions the oxidation

went on rather quietly. After standing for a day in the cold, small orange crystals having almost the same colour as $(NH_4)_2$ $[PtCl_4(OH)_2]$ were filtered off. Under the microscope these crystals proved to be quite homogeneous. The composition of this product corresponded to the formula $K_2[PtCl_4(OH)_2]$.

Found: Pt—43.25%, Cl_2—31.72%, K—16.95%.

Calculated: Pt—43.45%, Cl_2—31.57%, K—17.36%.

The salt $K_2[PtCl_4(OH)_2]$ is well soluble in water. After the evaporation of the solution on a water bath, potassium platinichloride, the tetrahydroxo-salt $KH[PtCl_2(OH)_4]$ and potassium chloride separated out as a rule.

Upon the action of hydrochloric acid on $K_2[PtCl_4(OH)_2]$ the two hydroxy-groups are substituted with chlorine with the formation of $K_2[PtCl_6]$. The oxidation reaction of potassium platinichloride and the subsequent changes of $K_2(PtCl_4(OH)_2]$ can be expressed as follows:

$$K_2PtCl_4 + HOOH \rightarrow K_2[PtCl_4(OH)_2]$$

$$3K_2[PtCl_4(OH)_2] \rightarrow K_2[PtCl_6] + 2KH[PtCl_2(OH)_4] + 2KCl.$$

The acid tetrahydroxo-salt $KH[PtCl_2(OH)_4]$ was isolated from its mixture with potassium platinichloride and potassium chloride after the evaporation of solutions containing $K_2[PtCl_4(OH)_2]$.

Found: Pt—51.80%, Cl_2—18.10%, K—9.97%.

Calculated: Pt—52.15%, Cl_2—18.95%, K—10.44%.

The acid corresponding to this salt was isolated already by Miolati and Wagner ([4]) from the dihydroxo-platinotetrachloric acid $H_2[PtCl_4(OH)_2]$.

In one of the experiments, after a protracted evaporation of the solution, besides potassium platinichloride, there separated a colloid-like precipitate of a dirty yellow colour which proved to be a dehydrated hexahydroxo-acid $H_2[Pt(PC)_6)$. The content of platinum in a sample dried at 120° was 72.24%.

Upon concentrating the mother solution after the separation of the main mass of $K_2(PtCl_4(OH)_2]$ under isothermal conditions in vacuo over sulphuric acid, the separation of K_2PtCl_4 $(OH)_2$ took place at first, and towards the end of the evaporation a mixture of potassium chloride with lightbrown crystals separated. On the basis of analytical data, the formula of an aquadihydroxo-salt could be ascribed to these crystals: $K[PtCl_3(OH)_2H_2O]$.

Found: Pt—50.00%, K—10.56%, Cl_2—26.58%.

Calculated: Pt—49.65%, K—9.92%, Cl_2—26.82%.

This compound is isomeric with the acid trihydroxo-salt $KH(PtCl_3(OH)_3)$. The determination of the molecular electroconductivity confirmed the correctness of the first formula.

v	2.000	4.000	8.000
μv	145,5	159	167 (determined by M. M. Yakshin)

The formation of $K[PtCl_3(OH)_2H_2O]$ should be represented as the hydrolysis of $K_2(PtCl_4(OH)_2$

$$K_2[PtCl_4(OH)_2] + H_2O \rightarrow K[PtCl_3(OH)_2H_2O] + KCl.$$

Oxidation of $K_2[Pt(NO_2)_4]$. The interaction of a 30% hydrogen peroxide with potassium platinotetranitrite also leads to the formation of several compounds. The main oxidation product is the dehydroxotetranitrite

$$\left[\begin{matrix} & OH & \\ NO_2 & & NO_2 \\ & Pt & \\ NO_2 & & NO_2 \\ & OH & \end{matrix}\right] K_2$$

This substance forms colourless crystals, well soluble in water.

Found: Pt—39.90%, N_2—11.70%, K—15.73%.

Calculated: Pt—39.72%, N_2—11.42%, K—15.91%.

Upon the evaporation on a water bath of the mother liquor containing an excess of hydrogen peroxide, at first there separated a large amount of white crystals that proved to be nothing else but potassium nitrate. A further concentration of the solution lead to the separation of beautifully formed lemon-yellow crystals that were like parallelepipeds. The analysis of a twice recrystallized product gave a ratio for Pt : K : N = 1 : 1 : 3.

Found: Pt—49.94%, K—10.41%, N_2—10.44%.

Number of NO_2-groups...3. The electroconductivity of this compound's solution was the nearest to a 3-ion electrolyte.

v	2.000	4.000	8.000
μ_v	313	337	359

On the basis of analytical and electroconductivity data the following formula should be ascribed to this salt: $KH[Pt(NO_2)_3OH]$. The somewaht high value for molecular electroconductivity for a 3-ion electrolyte can be explained by the considerable mobility of the hydrogen ion.

By continuing the evaporation of the mother liquor after the separation of $KH[Pt(NO_2)_3OH]$ we were able to isolate still another salt in the form of light-yellow needle-like crystals. The composition of this salt corresponded to the dihydroxo-tetranitrate $K_2[Pt(NO_3)_4(OH)_2]$.

Found: Pt—35.37%, N_2—10.63%, K—14.32%.

Calculated: Pt—35.21%, N_2—10.10%, K—14.10%.

The determination of the nitro-group with ammonium chloride gave negative results, confirming the formula accepted by us. Probably, the nitro-groups had been completely oxidized into nitrato-groups.

On the basis of the composition of the products obtained, the process of the interaction of potassium platonitrate with hydrogen peroxide can be expressed by the following scheme.

$$1.\quad K_2[Pt(NO_2)_4] + HOOH \rightarrow K_2[Pt(NO_2)_4(OH)_2].$$

2. The excess of hydrogen peroxide partly reduces the dihydroxo-tetranitrite to $KH[Pt(NO_2)_3OH]$:

$$K_2[Pt(NO_2)_4(OH)_2] + H_2O_2 \rightarrow KNO_2 + KH[Pt(NO_2)_3OH] + H_2O + O_2.$$

3. Potassium nitrite is oxidized to potassium nitrate.

4. The nitro-groups of the dihydroxo-tetranitrite are oxidized to nitrato-groups.

The oxidation of the first Reizet chloride $[(NH_3)_4Pt]Cl_2$ with dilute hydrogen peroxide leads to the dihydroxo-compound $(NH_3)_4Pt(OH)_2Cl_2$. This fact has been proved already by Chugajev and Khlopin. The interaction of the first Reizet chloride with 30% hydrogen peroxide proceeds somewhat differently with a partial migration of the chlorine ione into the inner sphere.

In a number of oxidations we always obtained good colourless crystals that could be easily recrystallized retaining their initial composition. We have not succeeded as yet to draw up a formula of this compound on the basis of its analysis.

Found: Pt—51.60%, N_2—15.00%, total Cl_2—21.24%, outside Cl_2—17.00%.

The first Reizet nitrate is smoothly oxidized into the dihydroxo-compound $[(NH_3)_4Pt(OH)_2](NO_3)_2$ with no by-products being formed.

The most interesting in this investigation on the oxidation of electrolytes of bivalent platinum with hydrogen peroxide was the possibility of obtai-

ning through oxidation the hexahydroxo-acid $H_2[Pt(OH)_6]$ in an absolutely pure state. This acid which can be used as an initial product for the preparation of most varied derivatives of bi- and tetravalent platinum, is usually obtained from salts which are formed through the action of alkalis upon $PtCl_4$ or H_2PtCl_6. The hexahydroxo-acid prepared in this way is always contaminated with alkali and salts which are very difficult to eliminate from the colloidal precipitate.

We oxidized Alexander base $[NH_2OH)_4Pt](OH)_2$, a compound that can be easily obtained from potassium chloroplatinate, with hydrogen peroxide under most varied conditions. and we always obtained the hexahydroxo-acid or products of its dehydration.

The hydroxylamine is oxidized to nitrogen and water and the place of the four hydroxylamino-groups is being taken by four hydroxyls. Simultaneously platinum is oxidized to a tetravalent form and the addition of two more hydroxyls takes place.

$$[(NH_2OH)_4Pt](OH)_2 + 3HOOH \longrightarrow H_2[Pt(OH)_6] + 2N_2 + 6H_2O.$$

The hydroxo-acid is obtained as a yellow product that darkens as it dehydrates.

The investigation of the preparation of derivatives of bi- and tetravalent platinum by way of substituting the hydroxyls of the hexahydroxo-acid for different groups is being continued.

Conclusions

1. The reaction of the oxidation of electrolytes of bivalent platinum with hydrogen peroxide $K_2[PtCl_4]$, $K_2[Pt(NO_2)_4]$, $[(NH_3)_4Pt]Cl_2$, $[(NH_3)_4 Pt](NO_3)_2$ was investigated. The main oxidation products are the dihydroxo-compounds of tetravalent platinum.

2. Upon the oxidation of $K_2[PtCl_4]$ and $K_2[Pt(NO_2)_4]$, next to dihydroxo-compounds. acid salts and aqua-salts of a lesser basity are formed.

3. Upon the oxidation with hydrogen peroxide of the Alexander base $[(NH_2OH)_4Pt](OH)_2$ the hexahydroxo-acid $H_2[PtCl_6]$ has been obtained.

Institute of General and Inorganic Chemistry.
Academy of Sciences of the USSR.

Received
20. V. 1939.

REFERENCES

[1] A. V. Babajeva, C. R. Acad. Sci. URSS, XXIII, No. 7 (1939).
[2] Л. А. Чугаев и В. Г. Хлопин, Изв. ин-та по изуч. платины, вып. 5 (1927).
[3] A. A. Greenberg a. F. M. Filinow, C. R. Acad. Sci. URSS, XVI, No. 1—2 (1927). [4] Miolati a. Wagner, ZS. f. phys. Chem. 28 (1899).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIV, № 2

CHEMISTRY

CHANGE IN THE DROP OF POTENTIAL AT THE AIR/SOLUTION INTERFACE AS RELATED TO THE AGE OF THE LATTER

By A. I. PANKRATOV

(Communicated by A. Frumkin, Member of the Academy, 20. V. 1939)

A paper by Klein and Lange (1) recently published deals with the time variation of the potential drop at the interface air/solution. According to the experimental data of the authors referred to, the drop of potential in the case of salt-solutions of medium and high concentrations varies with time from negative values to positive ones. In the case of a 5.46 *N* KBr-solution the drop of potential changes by over 300 millivolts in 100 min. Klein and Lange believe themselves to have discovered an effect of interface «maturing» due to a slow reorientation of water molecules or to some superficial process of crystallization. That the drop of potential at an air/solution interface varies with time has already been reported in literature, as for instance, in a paper by A. Frumkin (2), where it is pointed out, however, that a gradual contamination of the surface in question with surface-active materials from the surrounding air must be regarded as the principal cause of this change.

From studying the capacity of the electric double layer at the solution/metal interface, M. Proskurnin and A. Frumkin (3) arrived at a conclusion that traces of surface-active materials present in salts might be the principal source of contamination of the interface in the case of inorganic salt-solutions. These materials could be eliminated by a preliminary calcining of the salt. Adam (4) likewise pays attention to this circumstance. Therefore apart from protection of the interface under investigation from air contaminations due care should be taken to prepare solutions free from any surface-active materials in order to obtain proper results from measuring the drops of potential at the air/solution interface. The importance of this circumstance received added evidence from our own studies (5) of the mechanical and electrical properties of monomolecular layers on the surface of concentrated salt-solutions.

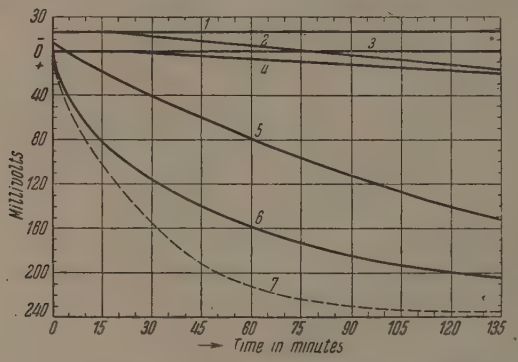
The present investigation suggested by A. Frumkin had for object definitely to establish the causes of the phenomenon observed by Klein and Lange; measurements of the drop of potential at the air/solution interface have been effected by the method of Guyot-Frumkin.

Schering's KBr was taken for the experiments. Thre eportions of 2 *N* KBr solution were prepared:one was obtained directly from commercial (uncalcined) salt and the other two from a heated salt; the latter two solutions were filtered through a glass filter. The salt was calcined till fusion in a platinum crucible. The water used for preparation of the first and second solutions

was twice distilled, the middle fraction only being collected during the second distillation. Water distilled with potassium permanganate was used for preparation of the third solution.

While checking constancy of the potential drop at the surface of the disc covered with polonium, use was always made of a freshly renewed surface of a 0.01 *N* KCl solution prepared from a calcined salt.

The experiment was run simultaneously with the first and second KBr solutions. The initial values of the potential drop for the first and second solutions were checked by renewing their surfaces repeatedly. Then the surfaces of both solutions were allowed to rest for 135 min. During this interval drops of potential at the surface of both solutions were measured (at first every 1—2 min. and afterwards every 10—15 min.). Throughout the experiment the surfaces of both solutions were not altogether inaccessible to the laboratory air in so far as the Faraday cage which protected the apparatus was not hermetically closed. Mean values from several series of measurements



are given in Fig. 1 in the form of curves where curve *6* and *2* show the change of the drop of potential at the interface between air and solutions prepared from uncalcined or calcined salt respectively.*

The drop of potential at the interface between the air and a solution prepared from calcined salt dissolved in water distilled with potassium permanganate was investigated in an analogous manner. These experiments were performed in such a way either to protect the surface of the solutions from the air dust as far as possible or, as it was the case in the preceding experiment, to leave it open to some extent to the action of dust.

Mean values of the drop of potential obtained in the latter case, fit in with the data obtained in the previous experiment and may therefore be represented graphically by curve *2* in Fig. 1. However, when the surface of the solution under investigation was carefully protected from dust, there was no change whatever in the drop of potential at the air/solution interface; the results of these measurements are represented by curve *1* in Fig. 1. To ascertain whether the drop of potential is liable to change under these conditions if they are upheld for a much longer time, we have carried out two experiments observing the behaviour of the surface during 4 hours in both cases; the drop of potential, however, remained constant and coincided within 0.3 millivolts with the value obtained with the same solution in a few seconds after the renewal of the surface. The straight line *3* and curve *4* in Fig. 1 represent the results of measurements at the air/0.01*N* KCl solution interface

* The zero point on the ordinate axis in Fig. 1 is the value of the drop of potentia at the interface air/0.01 *N* KCl solution when the surface of the latter is freshly renewed

in the cases of protected and unprotected surface, respectively. Curve *5* in Fig.1 was obtained with a KBr solution prepared from uncalcined salt, after 30 hours from the first experiment with the same solution (curve *6*) have passed. A somewhat higher course of curve *5* is accounted for by the fact that the experiment was brought about with a less contaminated solution, part of the surface-active materials being adsorbed at the surface of the solution. The latter was contained in a separate reservoir and the construction of the apparatus did not allow the surface-active materials adsorbed at the surface to get into the vessel, where the surface of the solution investigated was obtained. The dotted curve *7* represents the data of Klein and Lange ([1]) which they obtained in studying a 1.93 *N* KBr solution.

Conclusions

From the data obtained in studying the time variation of the potential drop at an air/solution interface, we arrive at the following conclusion: if the conditions of the experiment prevent an accumulation of surface-active materials at the surface of the solution, there is no change in the drop of potential during comparatively large intervals of time (lines *1* and *3* in Fig. 1). If not, the drop of potential changes to more positive values owing to a gradual accumulation of surface-active materials at the surface; these contaminations penetrate to the surface mainly from the solution and partly as a result of dust settling from the air which is in contact with the surface under investigation. The influence of both factors in the course of 135 min. effected, in our case, a change of 210 millivolts in the drop of potential of a 2 *N* KBr solution (curve *6*). The influence of the second factor alone changed the drop of potential by 35 millivolts (curve *2*) only. In the case of a 0.01 *N* KCl solution this change amounted to 20 millivolts (curve *4*). A smaller value of the change in the drop of potential for a 0.01 *N* KCl solution compared with a 2 *N* KBr solution is in agreement with our results of the studies of properties of monolayers on the salt-solutions, according to which the presence of KBr in the solution facilitates the spreading of monolayers over the surface. The surface-active contaminations which affect the drop of potential get into the solution with the dissolved salt and can be eliminated by a preliminary calcining of the latter. The degree of purification of distilled water had in our case no influence upon the change in the drop of potential.

Moscow State University.
Electrochemical Laboratory.

Received
23. V. 1939.

REFERENCES

[1] O. Klein u. E. Lange, ZS. Elektrochem. **8**, 562 (1938). [2] А. Фрумкин, Сб. работ ин-та им. Карпова, **4**, 74 (1925); ZS. physik. Chem., **116**, 485 (1925). [3] M. Proskurnina. A. Frumkin, Trans. Far. Soc., **31**, 110 (1935). [4] Adam, The Physics and Chemistry of Surfaces, Oxford (1930). [5] А. Фрумкин и А. Панкратов, ЖФХ, **12**, 541, 550 (1938); Acta physicochimica URSS, **10**, 45, 55 (1939).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIV, № 2

CHEMISTRY

CHANGE IN THE RATE OF SOLUTION OF COPPER DURING ANODIC AND CATHODIC POLARIZATIONS. DIFFERENCE-EFFECT. PROTECT-EFFECT

By N. D. TOMASHOW

(Communicated by Wl. A. Kistiakowsky, Member of the Academy, 14. IV. 1939)

In a number of papers ([1],[2],[3]) it was established that the phenomenon of the decrease in the rate of auto-solution of a metal, while it is being anodically charged with current (difference-effect) or cathodically charged with current (protect-effect), necessarily follows from the electrochemical mechanism of the solution of metals on the basis of the most general laws of electrochemistry (Kirchhoff's and Faraday's laws).

Hence, a very convincing proof of the purely electrochemical mechanism of the corrosion of copper ([4]) would be the establishment of the phenomena of the difference-effect and the protective-effect during the solution of copper in electrolytes.

We studied the kinetics of the solution of copper in 1 *N* and 0.1 *N* solutions of ammonium persulfate and in an 0.8 *N* solution of cupric chloride at room temperature 18—20° C by means of a gravimetric method. As is known, the solution of copper in persulfates proceeds without the formation of any gaseous or insoluble corrosion products. During the solution of copper in cupric chloride, however, a precipitate of Cu_2Cl_2 is formed, but, as was shown experimentally, this precipitate does not accumulate on the surface of the electrode, and therefore the method of continuous weighing, even though in this case it is somewhat less accurate than in the case of the persulfate solutions, could be employed to study the kinetics of the solution of copper in cupric chloride also.

A schematic drawing of the experimental set-up is given in Fig. 1.

In Table 1 there are given the fundamental results of these experiments. The data in columns 1, 3 and 5 of this table, i. e. the current strength in the circuit, the time of solution of a standard weighed amount, and the potential of the Cu-electrode with reference to a 1 *N* calomel electrode, were obtained directly by experiment.

The density of the polarizing current, the rate of solution, and the potential of the Cu-electrode with reference to a normal hydrogen electrode, were computed from these experimental data.

The curves shown in Fig. 2 were constructed on the basis of these data. The values of current densities of the anodic and the cathodic polarizations are plotted along the abscissae; the rates of solution in mg from 1 cm^2 of the surface per second are plotted along the ordinates (on the right), and against the potentials of the Cu-electrode with reference to a normal hydrogen electrode (upper left).

Table 1

Solution of Cu during Anodic and Cathodic Polarizations

Current strength in the circuit, in mA **	Current density in the Cu-sample, in mA/cm² **	Average time of solution of a standard amount of Cu per cm² in sec.	Average rate of solution of Cu, in mg/cm². sec.	Potential of Cu with reference to: a 1 *N* calomel electrode, in V	Potential of Cu with reference to: a 1 *N* hydrogen electrode, in V
		Solution 1 *N* $(NH_4)_2 S_2O_8$*			
0	0	458	0.0062	−0.0256	+0.270
+150	+37.5	205	0.0139	+0.226	+0.504
0	0	444	0.00642	−0.0245	+0.271
+100	+25	263	0.0108	+0.164	+0.450
0	0	445	0.0064	−0.024	+0.272
+ 50	+12.5	349	0.00817	+0.102	+0.388
− 20	− 5	950	0.0030	−0.070	+0.216
− 50	−12.5	∞***	0.0000	−0.292	−0.006
0	0	463	0.00615	−0.020	+0.276
		Solution 0.1 *N* $(NH_4)_2S_2O_8$*			
+150	+37.5	42	0.0134	+0.036	+0.322
+ 40	+10	111	0.0050	−0.001	+0.285
+ 11.2	+ 2.8	201	0.0028	−0.016	+0.270
0	0	267	0.00211	−0.028	+0.258
− 10	− 2.5	763	0.00074	−0.037	+0.239
− 17	− 4.25	∞***	0.0000	−0.862	−0.576
		Solution 0.8 *N* $CuCl_2$*			
0	0	613	0.00093	+0.011	+0.299
+ 75	+18.75	75	0.00763	+0.140	+0.426
+ 44	+11.0	120	0.00478	+0.070	+0.356
+ 30	+ 7.5	155	0.00370	+0.053	+0.339
+ 20	+ 5.0	217	0.00264	+0.040	+0.326
+ 10	+ 2.5	300	0.00191	+0.030	+0.316
0	0	619	0.00092	+0.015	+0.301
− 5	− 1.25	1.000	0.00058	−0.004	+0.282
− 10	− 2.5	2.000	0.00029	−0.027	+0.259
− 15	− 3.75	∞***	0.0000	−0.055	+0.231

The straight line *F*—*F* shows the dependence of the rate of solution upon the current density on the basis of Faraday's law if we assume that the copper upon solution formes cupric ions (the copper equivalent equals 31.79).

An analysis of the given data permits us to draw the following conclusions:

Note: * The standard amount of copper for experiments carried out in 1 *N* $(NH_4)_2S_2O_8$, in 0.1 *N* $(NH_4)_2S_2O_8$, and in 0.8 *N* $CuCl_2$ was equal to 2.85, 0.563, and 0.573 mg/cm² respectively, and corresponded to a decrease in weight of the sample equal to 10, 2, 2 mg respectively.

** A plus sign indicates anodic polarization of the copper electrode, a minus sign, cathodic polarization.

*** The ∞ sign, as used in this table, indicates that no change in weight of the electrode could be detected after 30 min. within the limits of sensitivity of the balance employed in this experiment, i. e. the decrease in weight was not greater than 0.0001—0.0002 g.

1. The rate of solution of copper in 1 N and 0.1 N solutions of $(NH_4)_2S_2O_8$, and also in an 0.8 N solution of $CuCl_2$, increases with an increase in the anodic polarization and decreases with an increase in the cathodic polarization.

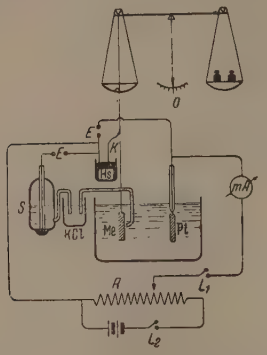


Fig. 1. A diagrammatic drawing of the set-up designed to determine the values of the difference- and protect-effects gravimetrically. *Me*, the metal sample (Cu) undergoing investigation, suspended on a copper thread insulated with Zapon-varnish from the pan of the analytical balance; *Pt*, electrode for polarization; *K*, a thin platinum wire which via a mercury bath (Hg), is used to make contact between the Cu-electrode undergoing investigation and the outside circuit; ε, points at which the Raps potentiometer was connected to measure potentials; *mA*, precision milliampermeter; *S*, calomel electrode.

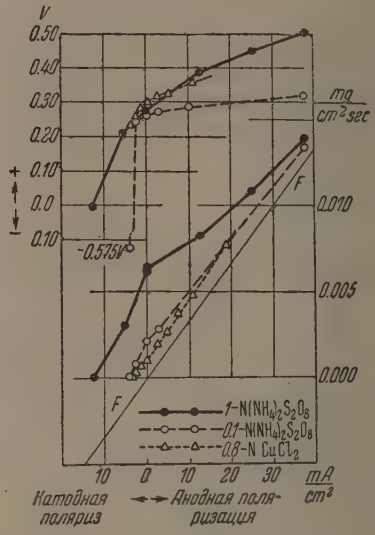


Fig. 2. Change in the rate of solution of copper (lower curves) and of the electrode potential (upper curves) as a function of the current density of the anodic and cathodic polarizations of the copper electrode.

2. The rate of auto-solution of copper* (in Fig. 2 this is represented by the intercept on the ordinate lying between the experimentally obtained curve and the theoretical straight line obtained according to Faraday) changes depending upon the character of the current charge imposed. For copper in solutions of 1 N and 0. 1 N $(NH_4)_2S_2O_8$ we observe a decrease in the rate of auto-solution when anodic polarization is imposed (positive difference-effect) and when cathodic polarization is imposed (positive protect-effect).

3. For copper in a 0.8 N solution of $CuCl_2$, we observe an increase in the rate of auto-solution when anodic polarization is imposed (negative difference-effect) and when cathodic polarization is imposed (negative protective-effect)**.

The negative difference-effect, even though it does not necessarily follow from the nature of the purely electrochemical solution of a metal, nevertheless does not contradict this type of solution, and points to the fact that the loosening of protective films on a metal during anodic polarization exerts the main influence here.

* That is, the rate of solution determined only by the operation of local microelements.

** However, inspite of the negative protective-effect which shows that the operation of local elements when cathodic polarization is imposed increases, the electrochemical protection of Cu in $CuCl_2$ solutions, contrary to Artamonov's (5) point of view, will take place, since here the main rôle is played by the fact that it is not hydrogen that will be evolved upon imposing a cathodic current on the Cu-electrode, but the more positive copper ions which are present in the solution and which, at a sufficiently large density, will completely compensate the solution of copper due to the operation of local elements.

4. The change in the potentials of the Cu-electrode as a result of cathodic polarization is much greater than it is during anodic polarization. Hence one may conclude that the areas of the copper surface which act as cathodes will have a tendency, during the operation of local elements, to stronger polarization than the areas which act as anodes (cathodic control of the rate of corrosion).

Table 2

Values of the Constants for the Difference-Effect (K_d) and the Protect-Effect (K_p) upon the Solution of Copper during Anodic and Cathodic Polarizations

Current density, in mA/cm²	Rate of solution corresponding to the current density (according to Faraday's law) from 1 cm² of the surface, in mg/sec.)	Total rate of solution (according to experimental data) from 1 cm² of the surface, in mg/sec.	Mean rate of autosolution from 1 cm² of the surface in mg/sec.	Difference-effect constant for the corresponding current density interval (K_d), in mg/sec. mA	Protect-effect constant for the corresponding current density interval (K_p) in mg/sec. mA
Solution 1 N $(NH_4)_2S_2O_8$					
−12.5*	−0.0042*	0.0000	0.0042		+0.00016
0.0	0.0000	0.0062	0.0062	+0.000178	
+12.5	0.0042	0.00817	0.00397	+0.000149	
+25	0.00835	0.01082	0.00247	+0.000128	
+37.5	0.0125	0.0139	0.0014		
Solution 0.1 N $(NH_4)_2S_2O_8$					
− 4.25	−0.0013	0.0000	0.0013		+0.00019
0.0	0.0000	0.00211	0.00211	+0.000032	
+37.5	0.0125	0.0134	0.0009		
Solution 0.8 N $CuCl_2$					
− 3.75	−0.00125	0.0000	0.00125		+0.000072
0.0	0.0000	0.00093	0.00093		
18.75	0.00618	0.00763	0.00145	−0.000028	

Table 2 contains difference- and protect-effect constants calculated from the experimental data for the solution of copper in the investigated solutions.

The experimental results lead to the conclusion that the processes of the solution of copper (and apparently of other noble metals) in oxidizing media may be interpreted on the basis of the general electrochemical theory of the corrosion of metals.

Colloidal Electrochemical Institute.
Academy of Sciences of the USSR.

Received
28. IV. 1939.

REFERENCES

[1] M. Straumanis, Korr. u. Metallsch., **14**, 2 (1938). [2] W. Müller u. Löw, ZS. f. Elektroch., **42**, 789, 830 (1936). [3] Н. Д. Томашов, ЖФХ, XII, 4, 414 (1938). [4] N. D. Tomashov, C. R. Acad. Sci. URSS, No. 7 (1939). [5] Б. П. Артамонов, Журн. хим. пром., XIV, № 7, 516 (1937).

Note. * A minus sign indicates cathodic polarization of the electrode, a plus sign anodic polarization.

** A minus sign indicates a process reverse to solution, i. e. the evolution of copper during cathodic polarization in accordance with Faraday's law.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIV, № 2

GEOLOGY

SECTION OF CAMBRIAN DEPOSITS OF THE NORTHERN MARGIN OF THE ALDAN PLATFORM AND OF THE ADJOINING FOLDED REGION

By J. K. DZEVANOVSKY

(Communicated by V. A. Obruchev, Member of the Academy, 19. V. 1939)

For some years the author was carrying on geological investigations within different areas of the right-hand bank of the Aldan river, chiefly in those parts where Cambrian sedimentary rocks are developed. During this time the author collected abundant material on the stratigraphy of the Cambrian of the above region which will be published in a separate work devoted to this subject. In this small paper the author gives description of the section of the Cambrian deposits constructed according to the author's observations with such supplements which may be made at present according to materials published by other students.

As a basis for the section shown in the following served data of the author's investigations made in the Central Aldan, Allakh-yun and Maya regions. Among other works the author used the data of A. A. Leontovich who worked in 1933/34 in the central part of the basin, and those of M. J. Stolyar on the Uchura-Chulba region. The section is constructed in accordance with abundant organic remains, determined by E. V. Lermontova (trilobites) and A. G. Vologdin (algae).

The Cambrian of the above region transgressively overlies the ancient crystalline base of the Aldan platform with the following series traced there in ascending order:

1. Mail series (Cm^1_1). Interbedding of grey ochrous-yellow quartz and quartzite-like sandstones with black argillaceous, brown marly and greenish-grey areno-argillaceous shales. At the base of the series a thin layer of basal arkose sandstone is lying with lenses of conglomerates, in which the pebbles consist of pre-Cambrian crystalline rocks. Thickness of the series 190—213 m.

2. Chelasin series (Cm^2_1). The lower portion is composed of chocolate-brown marls and grey tabular limestones, replaced higher up by grey massive algae (at the upper parts) limestones. The marls contain remains of small trilobites *Triangulaspis meglitzkii* (Toll.), *Pagetiellus lenaicus* (Toll.), in the grey limestones algae *Collenia plana* Vol., *Conophyton dzevanovskii* Vol. etc. Thickness of the series 80—100 m.

3. Nelkan series (Cm^3_1). The lower portions consist of dark-grey and black calcareous areno-argillaceous and argillaceous shales, replaced higher up by white, grey and ochrous-yellow quartzite-like sandstones.

According to A. A. Leontovich, this series is terminated by a horizon of grey oölite limestones, overlying the sandstones. Thickness of the series 100—150 m.

4. Upper Maya series (Cm^4_1). Isolated by the author according to A. A. Leontovich's materials. Consists of white, pinkish and grey crystalline limestones porous and dolomitized. Thickness of the series not less than 100—150 m.

5. Aldan series (Cm^5_1). Isolated by the author according to M. J. Stolyar's materials; consists of white crystalline limestones in the lower part with quartz sandstone bands, and in the upper half of interbedded limestones and red marls with *Archaeocyathus* fauna: *Protopharestra* sp., *Spirocyathus* sp., *Coscinocyathus corbicula* Born., *Subradiatus* Vol. etc. Thickness of the series above 200 m.

The above section of the lower portion of the Cambrian beds is characteristic only of the northern and eastern marginal parts of the platform. In its central portions the lower series are absent and here the upper series are lying directly on the denuded surface of the platform.

The above five series, about 820 m thick, compose a vast area between the Uchur and Maya rivers, bordering as shown above the crystalline base of the Aldan platform from the north and the east. Owing to the subsidence of the latter in the direction from south to north, an increase is observed in this direction of the limestone series at the expense of a thickness of the lower series composed of terrigene sediments. This subsidence of the platform and of the sediments is overlain within the above-mentioned direction results in the appearance of the upper members of the section which are given below.

6. Kulakh series (Cm^6_1). Composed at the base of white and dark-pink quartzite-like sandstones, higher up replaced by chocolate-brown algae, greenish-grey finely tabular and grey massive limestones, the upper portion of which is covered with sandstones and black argillaceous shales with trilobite fauna *Protolenus dzevanovskii* sp. nov., *Brachiopoda—Botsfordia coelata* Hall. The following algae have been encountered here. *Collenia undosa* Wal., *Collenia bobini* Vol. etc. Thickness of the series 130 m.

7. Mutulin series (Cm^1_2). Composed of finely and coarsely tabular greenish-grey limestones, higher up interbedded with tabular brownish-red marls. In the middle portion a layer of grey brecciated limestones is enclosed (up to 20 m in thickness), while higher up it is composed of greenish-grey compact arenaceous limestones with ripple-marks at the bedding planes.

In the lower finely tabular limestones *Goniagnostus nathorsti* Brögg., *Ptychagnostus punctuosus* (Ang.) and others are enclosed, and in the upper arenaceous limestones—in their base—*Ctenocephalus obesus* sp. nov., and somewhat higher still *Centropleura* (*Clarella*?) *oriens* sp. nov. Thickness of the series 120 m.

According to Lermontova, between the layers with *Agnostida* fauna referred to the upper portions of the *P. davidis* zone, and the shales with *Protolenus* (upper part of the Lower Cambrian) one more considerable packet of layers must be lying, corresponding to the lower portions of the *P. davidis* zone. Its absence in our section may be explained either by a considerable decrease in the thickness of the presumptive layers, or by their absence from the section owing to a repeated shallowing of the Middle Cambrian sea, spreading over some areas of the region.

As to the fauna of the upper part of this series, it corresponds (according to E. V. Lermontova) either to the upper portions of the *P. davidis* zone or to the lower portions of the *P. forchhammeri* zone.

8. Dzhyunyukan series (C^2_2). Begins with a horizon of

brownish-red, locally yellow limestones, containing abundant remains of calcareous algae. Higher up they are replaced by dark-grey compact massive limestones, in the upper parts of the series passing to dark-grey, almost black calcareous-arenaceous shales. Of the algae *Collenia undosa* Wal., *Conophyton sakharensis* sp. nov. and other are encountered here. Thickness of the series 95 m.

The sediments of the above two Middle Cambrian series are developed throughout the Yudoma-Maya and the Yudoma-Allakh-Yuna watershed where layers of the lower series appear from under the cores of anticlinal folds. Along the left-hand bank of the Maya river, owing to the uprise of fold axes towards the platform and their dying out, the sediments of these series are replaced, as shown above, by Lower Cambrian layers traced from here to the ancient crystalline base of the platform.

In some areas of the right bank of the Aldan river, in the zone of plicative dislocation manifestations, above the layers of the *P. forchhammeri* zone, younger beds are referred to the Upper Cambrian. Their appearance has been marked already at the Yudoma-Maya watershed, and the Upper Cambrian age is evidenced by the trilobite fauna detected by the author in the *Dzhyunyukan* river basin. In contradistinction to the layers of the first two divisions of the Cambrian, they do not form anywhere continuous fields, but appear as thin belts, jammed in the cores of synclinal folds. The composition of these beds is as follows.

9. Darsu series* (Cm_3). Begins with a horizon of light yellowish-white ochrous sandstones, passing to more compact quartzite-like sandstones. In the lower portions contains bands of grey tabular calcareous sandstones with ripple-marks at the bedding planes. The upper portion of the series consists of uniform beds of greenish-grey finely tabular limestones, enclosing in their middle portion a layer of brecciated limestones. In the finely tabular limestones remains of *Lotagnostus grandes* sp. nov. and others have been encountered. Thickness of the series 145 m.

The above-mentioned fauna of the Darsu series closely resembles the *L. asiaticus* Strand. from the Tyan-Shan, characterizing here rather high horizons of the Upper Cambrian.

Thus, aside from the sediments of the Lower and Middle divisions of the Cambrian widely spread throughout the right-hand band of the Aldan, we have here a complete section of the faunistically characterized upper division. The total thickness of the deposits of all three divisions of the Cambrian is 1160—1300 m at the first approximation. To show here the sections of other authors used for this work is beyond the scope of the present paper. We shall restrict ourselves to a summarized stratigraphic table of these sections giving thus such an interpretation which in our opinion is most suitable to the general stratigraphic scheme of the Cambrian deposits of the region under description.

Gold Prospecting Trust.
Moscow.

Received
20. V. 1939.

* Called thus from a river of the same name (Dzhyunyukan river basin) where the rocks composing it have been first encountered.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIV, № 2

GEOLOGY

A CONTRIBUTION TO THE STRATIGRAPHY OF THE CAMBRIAN OF THE MAYA RIVER BASIN

By V. N. MAKHAEV

(Communicated by M. A. Usov, Member of the Academy, 22. V. 1939)

In summer 1937 the author was making geological survey of the Gerb river basin (a left-hand tributary of the Yudoma river); in the course of this work he followed the route along the Yudoma river from the mouth of the Gerb river to the Maya river, and along the Maimakan river (a left-hand tributary of the Maya river) from the mouth to the upper reach of the right-hand tributary of the Maimakan, the Uodolà river.

Privious to this, no explorations whatever were made in the Gerb river basin. Along the Yudoma river up-stream of the mouth of the Gerb river geological survey has been made by D. F. Baikov at a scale 1 : 100 000. Along the Uodolà Eng. Ballod searched for alluvial gold. In summer 1937 the lower course of the Maimakan river was surveyed by Alekseychik, geologist of the Leningrad Petrol Institute; the results of these works are still unpublished.

In the following a description is given of the stratigraphy of the Cambrian of the Gerb and Yudoma rivers, comparison being made between these and Cambrian sections of other regions of the Central Siberian platform. Paleontological determinations belong chiefly to E. V. Lermontova, geologist at the All-Union Geological Scientific Research Institute, to whom acknowledgements are due for her valuable suggestions.

Lower Cambrian of the Yudoma River. Ustyudoma beds Cm_1^a. The most ancient sediments of the Lower Cambrian along the Yudoma river have been traced in the sections near the mouth. They consist of alternating beds of grey limestones with small inclusions of bitumens, bituminous dolomites, arenaceous limestones, arkose sandstones, quartz sandstones and dolomitized limestones with a total thickness of 65 m.

The Taima beds Cm_1^b overlying the Ustyudoma beds show the following section in ascending order: a) Yellow argillaceous limestones with ochrous stains and «flows»; along the bedding planes there are winding worm-holes; occasionally considerable accumulations of thick-valve *Hyolithes*, buried in a tanatocenotic environment; thickness up to 35 m. b) Black (odoriferous) bituminous marls with bands of black bituminous clayey shale enclosing veins of bituminous calcite and marcasite nodules. Along the bedding planes abundant *Protolenus* sp. nov., *Botsfordia* caelata (Hall.), *Kooteniopsis jakutensis* Lerm., *Hyolithes* sp. have been detected at the right-hand bank of the Yudoma river 15 km up-stream of the mouth.

Lower Cambrian of the Gerb River. The Taima beds Cm_1^b have been established along the Taima river (mouth) and along the left-hand bank of the Gerb river, 2.5 km down-stream of the mouth of the Taima river. Their section in ascending order: a) Yellow coarsely tabular limestones with a knobby surface, ripple-marks and sinuous worm-holes along the bedding planes; thickness 15 m. b) Black (odoriferous) bituminous shales and marls with veinlets of bituminous calcite, with inclusions of marcasite and small siliceous concretions; thickness 12 m. *Protolenus* sp., *Botsfordia caelata* (Hall.), *Hyolithes* sp.

Analyzing the fauna from the Gerb and Yudoma rivers, it should be noted that the presence of *Protolenus* sp., *Kooteniopsis jakutensis* Lerm. in the sediments of the Taima beds points to the appurtenance of both Taima and Ustyudoma beds to the upper division of the Lower Cambrian (*Protolenus* zone). The alternation of arkose sandstones, dolomites, quartz sandstones and limestones, the conditions of fauna earthing and the presence of ripple-marks, testify to a shallow-water environment of sediment accumulation.

Provisorily we refer to the lower part of the Ustyudoma beds the arkose sandstones, dolomitized limestones and argillaceous limestones of the lower course of the Maimakan river where they overlie the crystalline pre-Cambrian.

As to its faunistical and lithological characters, the upper part of the Lower Cambrian of the Gerb and Yudoma rivers may be well compared with the section of the Lena river basin (Cm_1^b Cm_1^c. Cm_1^d) made by D. K. Segebart (1).

Middle Cambrian of the Gerb River. The Lower Gerbian beds Cm_2^a overlie the Taima beds. They contain in ascending order: a) Packets (0.20—0.80 m) of grey, pink, chocolate coloured «lumpy» marls, interbedded with grey-green finely-tabular limestones. The total thickness is up to 70 m. Under the name of «lumpy» marls we understand here marls that are filled with small irregular lumps of limestone (occasionally silicified). In the upper portion considerable accumulations are frequenlty observed of impressions of the leading Middle Cambrian algae: *Bosworthia sibirica* nov. sp., *Ptychagnostus punctuosus* (Angelia), *Goniagnostus nathorsti* Brögger *Enetagnostus kjerulfi* (Brögg.), *Diplagnostus planicauda* (Ang.) b) Greenishgrey, sometimes chocolate-coloured finely-tabular argillaceous limestones, interbedded with occasional greenish marls. At the bedding planes impressions of the tallom *Bosworthia* are very frequent. The abundant accumulation of *Bosworthia* resulted in the formation of thin bituminous marl bands. The total thickness is 120 m. They contain: *Enetagnostus kjerulfi* (Brögg.), *Diplagnostus planicauda* (Ang.), *Goniagnostus nathorsti* (Brögger), *Hypagnostus truncatus* (Brögger), *Scandognostus repandus* Holm et West., *Clavagnostus repandus* (?) Holm et West., *Triplagnostus lundgreni* Tullb., *Phalacroma hyperborea* Holm et West. *Grandagnostus glandiformis* (Ang.), *Ptychagnostus punctuosus* (Ang.), *Agnostus* sp. nov. c) Grey tabular limestones, dolomitized limestones with sparce bands of pink limestones, with bands of argillaceous limestones. Thickness 70 m. *Centropleura hicksi* (?), *Anopolenus henrici* (Salter). All these horizons may be easily traced along the Gerb river, down-stream from the mouth of the Taima river.

Middle Gerbian beds Cm_2^b. The section in ascending order is as follows: a) Grey finely-tabular limestones, resembling the «c» limestones, but with 5—8 m bands of quartz-like sandstones and dark bituminous limestones. Total thickness 80 m. b) Massive light-grey arenaceous limestones with sparce drop-like inclusions of bituminous substance. In the lower part two bands of quartzite-like sandstones (Ambardakh region). Thickness 90 m. c) Interbedded bluish-grey oölitic limestones with pink tabular limestons

(packets 3—10 m thick), containing *Conophyton* sp., *Collenia frequens* Wal. Total thickness 60 m.

Higher up beds of light grey oölitic limestones are lying with a conglomerate horizon at the base. A microscopic examination of thin-sections showed them to contain algae which testify to the Lower Silurian age of the beds (fam. *Dasycladaceae*).

The total thickness of the Middle Cambrian sediments of the Gerb river is 490 m.

Middle Cambrian of the Yudoma River. Beds with *Protalenus* are concordantly overlain by Lower Gerbian beds, represented by greenish-grey, finely-tabular, occasionally breccia-like limestones, green, chocolate-coloured and variegated marls (sometimes lumpy marls). In the marl beds thin, bituminous, argillaceous shales are observed formed by an accumulation of algae *Bosworthia sibirica* Mach. (exposure 4.5 km down-stream of the Gerb river mouth). Total thickness over 160 m. The beds contain: *Cotalagnostus altus* Grönw., *Hypagnostus truncatus* (Brögger), *Enetagnostus kjerulfi* (Br.), *Diplagnostus planicauda* (Ang.), *Goniagnostus nathorsti* (Brögg.), *Trilobita*.

Thus, higher horizons of the Middle Cambrian have not been observed along the Yudoma river within the area from the Yudoma river mouth up to the Gerb river mouth.

Conclusions

The boundary between the Lower and Middle Cambrian along the Gerb and Yudoma rivers is found basing on faunistic data to pass beyond the upper boundary of black marls and shales with *Protolenus*. The Middle Cambrian fauna of the Gerb and Yudoma rivers is analogous to that of the *Paradoxides davidis* zone in Scandinavia and England. Layers containing this fauna may be juxtaposed to the sections of the Middle Cambrian of the Aldan and Maya rivers.

Gold Prospecting Trust.
Moscow.

Received
23. V. 1939.

REFERENCES

[1] Д. К. Зегебарт, Бюлл. Моск. о-ва исп. природы, отд. геол., XIV (3) (1936).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIV, № 2

MINERALOGY

DATOLITE FROM THE SOGANLUG TALUS IN THE NEIGHBOURHOOD OF THE TOWN OF TBILISSI

By G. V. GVAKHARIA

(Communicated by A. E. Fersman, Member of the Academy, 10. V. 1939)

The object of the present paper is to try and interpret the interrelation of the crystal forms of datolite from the Soganlug talus and the conditions of their formation. The investigations were made in connexion with the study of the post-volcanic processes of Georgia and the formation of minerals connected with them. As the material for the present work served the collections made by the author during the time from 1930 to 1939, and two crystals kindly placed at the author's disposal by G. G. Laemmlein.

The Soganlug talus is lying along the right-hand bank of the Kura river 6 km eastward from the town of Tbilissi on the high road Tbilissi-Luxenburg.

The geology of the region is as follows: the Soganlug talus is the eastern termination of the Telet range and represents the southern limb of the Telet anticline, having a latitudinal strike. Further east the axis of the anticline is submerged under younger sediments.

This part of the Telet range is composed of schistose, marls and clayey shales exposed only in ravines.

These are overlain by a dominating series of «conglomerates of complicated bedding», isolated by Abich ([5]) and consisting of clayey shales, limestones and volcanic rock blocks cemented by volcanogene tuffaceous material. The age of this series is established by recent works as the Lower Lutetian and its origin is explained by volcanic activity ([1, 5]).

The numerous outcrops of igneous rocks, in particular also the outcrops of the andesite-basalt of Soganlug producing an extensive talus, are to be referred to subaqueous volcanic activity, but in all probability to that of a considerably later period. The andesite-basalt of the Soganlug outcrop has been studied by Prof. G. M. Smirnov. He has given the chemical analysis of the rock and a geological-petrographic description ([2]). With the andesite-basalts and partly with the tufogene a great variety of mineral species is connected, first established by Prof. A. A. Tvalchrelidze ([3]).

The mineral formations of the talus are connected: 1) with the filling of numerous hollows chiefly in the peripheral porous part of the andesite-basalt outcrop, 2) with tuffs and 3) with the contact zone of the lava and the tuffaceous beds, being arranged very frequently at the near contact of tuffs themselves. All these minerals, either filling the hollows in the form of geods or segregated in small fractures, originate from solutions connected with the post-volcanic stage of solidification of andesite-basalts.

There is no doubt that the general geological conditions of their formation are somewhat different both with regard to temperature and the chemical

processes in tuffs, andesite-basalts and at the contacts. This difference manifested itself in the various types of datolite crystals established in geodes of different rock types.

Four types of crystals have been distinguished which were found to be fully dependent on the geological environment, in particular on the petrographic character of the enclosing rock.

Of the I type of datolite crystals (Fig. 1) a clearly manifested prismatic zone is characteristical with faces: *c*, *g*, *m*, *b*, *T*, as well as a perfect development of the base pinacoid *a*. Of the oblique faces *f* is the one best developed. Aside from the above faces *W*, *M*, *O*, *q* and ξ have been also established. Crystals of this type are characteristic of geodes concentrated in tuffs. Datolite is associated with leonghardite, mesolite and thompsonite. Leonghardite in a thin crust underlies the geodes and fissure-walls. Mesolite has been isolated in small quantities in some geodes after leonghardite. Mesolite is followed by thompsonite, the quantity of which is 5—6 times greater than that of leonghardite and mesolite. On the thompsonite well-developed large datolite crystals up to 1.5 cm are seated. Their quantity in some hand-specimens is even higher than that of thompsonite.

Type II of datolite crystals is characterized by a barrel-shaped cubic habit, slightly elongated along the *y*-axis. Most developed (Fig. 2) is the *X*-face and almost the same is observed of the face ϰ. Very characteristic of this crystal type are the faces μ and ε˙ forming parallel edges between themselves and with the face ϰ˙ which enables crystals of this type to by easily distinguished. The prismatic zone has a subordinate development. At this crystal type combinations of the following forms have been established: *x*, ϰ˙, μ, ε˙, *a*, *s*, *n*, *M*, *b*, *m*, *g*, *t*, *c*. Crystals of this type are characteristic of geodes and andesite-basalts for datolites of the first generation. On the basis of a study of a number of hand-specimens the sequence of isolation and the mineral association in geodes connected with andesite-basalts may be represented as follows: delessite, dark green prehnite, chalcedony or calcite form a border around the geode, followed by slightly yellowish datolite with type II crystals; on this datolite is again prehnite of a light-yellow colour, on which transparent and colourless type III datolite crystals are seated.

Type III of datolite crystals is characterized by a clearly pronounced finely tabular form, depending on the high development of the *X*-face and an almost absolute absence of the prismatic zone (the single very slightly developed *C*-face not being considered). The absence of the base pinacoid and the prismatic zone produced great difficulties with regard to measurements. The author arbitrarily accepted the *X*-face as a base pinacoid and by means of rotations on the Wulf plat the mineral was given a suitable position (Fig. 3). At this type of crystal the following forms have been found: *X*, *c*, *a*˙, Λ, *n*, *o*, *A*. Crystals closely resembling this type, tabular with a perfectly developed face *X*, have been described by Farrington from Guanajuato (7). Nearly similar paragenetic relations of prehnite and datolite are characteristic of the Toggiano deposit which are connected with hollows in diabases (8).

Type IV of datolite crystals is characteristic of geodes located in the andesite-basalt and tuffogene contacts. The crystals of this type are strongly elongated along the *y*-axis and are characterized by a dominating development of the δ and ξ edges (Fig. 4). At crystals of this type combinations of the following simple forms have been established: δ˙, ξ, *a*, *c*, *t*, *h*, *s*, *b*, Σ, *A*˙, *R*˙, J˙. With datolite of this type are associated: quartz in the form of thin film, underlying the geodes, delessite, following the quartz also as a thin film, occasionally in some specimens calcite, then a greenish-yellow prehnite, and finally datolite, on which laminae of specular iron crystals are seated.

Datolite is partly covered with stalactites or small pointed calcite scalenohedrons. In some specimens late malachite and early mesolite have been encountered.

Thus, we have four clearly pronounced types of datolite crystals, closely dependent on the conditions of their formation.

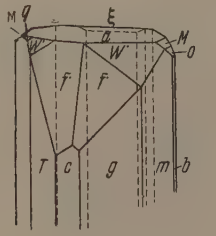


Fig. 1.

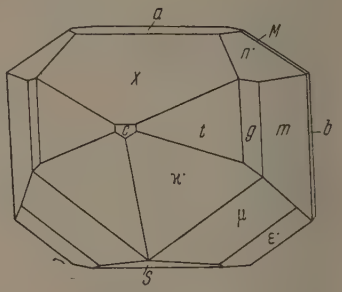


Fig. 2.

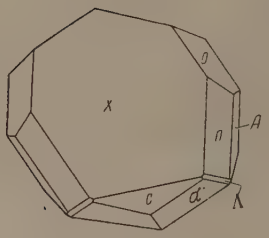


Fig. 3.

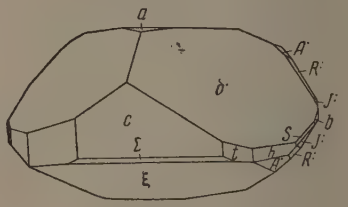


Fig. 4.

Table 1

Nos	Goldschmidt		Dana		Nos	Goldschmidt		Dana	
	Symbols	Indices	Symbols	Indices		Symbols	Indices	Symbols	Indices
1	*a*	(001)	*a*	(100)	14	0	(021)	0	(120)
2	*b*	(010)	*b*	(010)	15	Λ°	(111)	Φ	(112)
3	*c*	(100)	*c*	(001)	16	ε°	($\bar{1}11$)	ε	($\bar{1}12$)
4	*g*	(110)	*g*	(012)	17	n°	(122)	*n*	(111)
5	*m*	(120)	*mx*	(011)	18	q°	(113)	*q*	(312)
					19	*n*	(131)	—	(132)
6	*t*	(320)	*t*	(013)	20	W°	(211)	*W*	(114)
7	*T*	(250)	New form, author's symbol						
8	*S*	(140)	*s*	(021)	21	*J*:	($\bar{1}$·12.4)	*J*	($\bar{2}61$)
9	*h*	(340)	*h*	(023)	22	A°	($\bar{1}43$)	New form, author's symbol	
10	*x*	(101)	*x*	($\bar{1}02$)	23	α°	($\bar{2}21$)	α	($\bar{1}24$)
11	ξ	($\bar{1}01$)	ξ	($\bar{1}02$)	24	ζ°	(522)	ζ	(115)
12	ε	($\bar{1}03$)	ε	($\bar{3}02$)	25	μ°	($\bar{2}11$)	μ	($\bar{1}14$)
13	*M*	(011)	*m*	(110)	26	*s*	(344)	New form, author's symbol	
					27	*A*	(984)	Do	
					28	*R*:	($\bar{1}84$)	Do	

The measurements of the crystals were made on a two-circle Goldschmidt goniometer. The mounting and designations have been adopted according to Goldschmidt, only for new faces the author's own designations have been

introduced. Twenty eight forms have been established, summarized in Table 1. In this table the indices and designations by letters are given employed in Goldschmidt's and Dana's designs.

In the figures accompanying this paper Goldschmidt's symbols have been adopted. Table 1 shows that new faces have been established at the Soganlug crystals, not included in Goldschmidt's and Dana's above cited summarized data ([6,9,10,11])

T (250) $\varphi = 32°46'$ $\rho = 90°$; R: (184) $\varphi = 10°35'$ $\rho = 52°05'$
A' (143) $\varphi = 20°28'$ $\rho = 42°07'$;
A (984) $\varphi = 61°06'$ $\rho = 68°52'$; S' (344) $\varphi = 49°37'$ $\rho = 44°12'$.

Physical Properties. Colourless, occasionally light-yellow to dull. $2V$ from two emergences $= (-)72°$. $N_{g_1} = 1.666 \pm 0.002$; $N_{p_1} = 1.626 \pm 0.002$; $N_g - N_p = 0.040$. Orientation of the optical ellipsoid $N_g \wedge C = 3°$, $N_m = b$.

In the thin-section two imperfect cleavage planes or joints are to be seen, intersecting each other at an angle of 40°.

	For the first		For the second
	N_g	$-62°$	N_g $-37°$
P_1	N_m	$-81°$	P_2 N_m $-60°$
	N_p	$-30°$	N_p $-37°$

Optical orientation and other physical properties coincide with literary data. The chemical properties are those usual for datolite.

The chemical composition of the Soganlug datolite closely approaches the theoretical. For analysis pure large crystals were taken from a geode in the andesite-basalts (Table 2).

Table 2

Analyzer I. M. Shumilo

		Molecular proportions	Quantity of molecules
SiO_2	36.94	0.6155	2.023
TiO_2	none	—	—
Al_2O_3	0.31	0.0003	
B_2O_3	21.15	0.3037	1
Fe_2O_3	0.09	0.00006	
P_2O_5	none	—	—
CaO	35.17	0.6272	2.062
MnO	0.02	0.0002	
MgO	none	—	—
K_2O	traces	—	—
Na_2O	traces	—	—
$H_2O < 110°$	0.10	0.3410	1.121
$H_2O > 110°$	6.06		
Total	99.84		

Thus, the composition of the Soganlug datolite may be expressed by the formulae:

$2\,CaO \cdot B_2O_3 \cdot 2SiO_2 \cdot H_2O$ или $CaB_2\,Si_2O_8 \cdot Ca\,(OH)_2$

The presence of datolite in the post-volcanic formations of the Caucasus is a characteristic geochemical feature bearing testimony to the high B_2O_3 content of these rocks.

Department of Crystallography,
Mineralogy and Petrography,
State University
and Georgian Industrial Institute.

Received
9. V. 1939.

REFERENCES

[1] И. В. Качарова, Бюлл. ин-та Грузии, II, вып. 1 (1936). [2] Г. М. Смирнов, Изв. Зак. географ. о-ва, XXIV (1916). [3] А. А. Твалчрелидзе, Путевод. по Тифлису и его окрестн. (1925). [4] В. Г. Чикоидзе, Геология и петрография эффузивов окрестн. Тбилиси (1937) (рукопись). [5] Abich, Geolog. Forsch. in d. Kaukas. Ländern, III (1887). [6] E. S. Dana, The System of Mineralogy (1920). [7] Farrington, Amer. Journ., 5 (1898). [8] P. Gallitelli, Atti della Soc. Toscana di scienze naturali, XXXVIII (1928). [9] V. Goldschmidt, Kristallogr. Winkeltabellen (1897). [10] V. Goldschmidt, Atlas d. Kristallformen (1923). [11] R. Görgey u. V. Goldschmidt, ZS. f. Krist. u. Miner., 48, H. VI (1911).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIV, № 2

MINERALOGY

DATOLITE FROM THE VOLCANIC GROUP OF THE KARADAG IN THE CRIMEA

By M. N. SHKABARA and E. A. STURM

(Communicated by A. E. Fersman, Member of the Academy, 10. V. 1939)

During repeated visits to the Karadag the authors have found datolite, aside from the Andesite sopka, also as the Karagach range in the primary occurrence of andesites, in the form of beautifully developed water transparent crystals, in some cases up to 0.5—1 cm in cross-section. Both, in the first and in the second deposit datolite has been isolated in geodes and fissures, but the deposits differ from each other by the character of segregations, paragenesis and habit of the crystals.

The crystals found by us differed from those described for the Karadag by their habit, colour and transparency and by the character of the filling of cavities as well, which made us to subject them to goniometric and chemical investigation.

Andesite sopka named by some authors «Razvalka», «Kamenolomnya», «Syry-Kaya on the Tepsene» is located at the northern part of the Karadag igneous group near the village of Koktebel.

Datolite is usually encountered here in geodes and filled fractures, constituting their principal component in association with calcite, quartz, delessite and analcime. For this deposit the following sequence of these minerals is shown:

According to S. P. Popov: 1. a) delessite, b) quartz, frequently greenish, sometimes, amethyst and datolite; 2. a) delessite; b) quartz; c) analcime; d) datolite; 3. a) a solid calcite crust; b) a crust of radial datolite(botriolite). Aside from this the authors have established: 4. a) well developed calcite crystals of the first generation (up to 2—3 cm); b) datolite in the form of a solid mass or crystal intergrowth; c) a crust or crystals of calcite of the second generation.

The datolite of the andesite sopka is mostly of a violet colour. Well-developed crystals are rare. Table 1 gives the chemical analysis of the violet datolite crystals.

Karagach range. The datolite of this deposit is confined to the fractures of andesite rocks, it fills these fractures with watery transparent crystals 0.2—1 cm in diameter. A study of the specimens shows the following sequence of isolation of the minerals encountered in association with datolite: 1) calcite, 2) pyrite, 3) thompsonite and 4) datolite. The chemical analysis of the watery-transparent datolite crystals is shown in Table 1.

At the Karagach range, aside from andesites, datolite is encountered in the form of dull barrel-shaped crystals in tuffs in association with calcite.

Table 1

Analysis of violet crystals of the datolite of the Andesite sopka				Analysis of the watery transparent datolite crystals from the andesite Karagach range			Analysis of barrel-like datolite crystal from the tuffs of the Karagach range		
Content %		Mol. proport.	Mol. quantity	Content %	Mol. proport.	Mol. quantity	Content %	Mol. proport.	Mol. quantity
SiO_2	36.83	614	1.98	37.08	631	2.04	37.55	627	2.03
$Al_2O_3+Fe_2O_3$	0.45			0.29			0.33		
CaO	35.52	634	2.06	35.41	633	2.06	35.39	532	2.06
MgO	0.19	5		0.17	4		0.26	6	
B_2O_3	21.66	311	1	21.51	309	1	21.50	309	1
H_2O	6.12	340	1.09	6.00	333	1.08	5.86	325	1.05
Total	100.77			100.56			100.89		

Formula: $2SiO_2 \cdot 2CaO \cdot B_2O_3 \cdot H_2O$.

Table 2

Goldschmidt		Dana		Goldschmidt		Dana		Goldschmidt		Dana	
a	001	*a*	100	*p*	141	Author's symbols		$\varkappa$	$\bar{1}41$	Author's symbols	
b	010	*b*	010	*m*	126?	Do		ν	$\bar{1}22$	ν	111
c	100	*c*	001	*e*	139	Do		—	$\bar{1}21$	—	122
m	120	m_x	011	*M*	011	*m*	110	ε	$\bar{1}11$	ε	112
t	320	*t*	013	*O*	021	*O*	120	Σ	$\bar{1}03$	Σ	302
g	110	*g*	012	γ	221	γ	124	μ	$\bar{2}11$	μ	114
σ	210	σ	014	*W*	211	*W*	114	α	$\bar{2}21$	α	124
λ	111	Φ	112	*U*	342	*U*	123	*i*	$\bar{3}42$	*i*	123
x	101	*x*	102	*H*	3.7.21	Author's symbols					
n	122	*n*	111	*L*	322	*L*	113	ω	$\bar{3}11$	ω	116
ψ	104	ψ	201	ζ	$\bar{5}22$	ζ	115	$\varkappa$	$\bar{5}22$	$\varkappa$	115
Q	121	*Q*	122	ξ	$\bar{1}01$	ξ	$\bar{1}02$	π	$\bar{4}73$	Author's symbols	
								ϑ	$\bar{7}13$	Do	
								μ	$\bar{7}43$	Do	
								∂	$\bar{1}3.5.5$	Do	

The analysis of these crystals is also shown in Table 1. The above paragenesis and sequence of isolation of datolite and the minerals associated with it shows that the datolite is crystallized everywhere after calcite and zeolite. As seen from a comparison of our three analyses of datolites, their composition is very similar, closely approaching the theoretical formula: $HCaBSiO_5$.

For a crystallographic description crystals have been taken from small fissures of andesites of the Karagach range, seated either on calcite or directly on thompsonite lamellae. All the crystals have been subdivided into four groups according to combinations. Out of every group one sample with good

Table 3

Forms	Number of crystals	Number of faces	Measured		Calculated	
			P	S	P	S
κ $\bar{1}41$	1	1	69°77	21°30	69°55	338°30
ε 139	1	1	14°50	33°15	14°45	151°53
p 141	1	2	69°55	262°23	69°50	21°01
λ $\bar{4}73$	1	1	63°30	101°15	63°25	219°53
μ $\bar{7}43$	1	1	69°25	127°10	69°25	245°48
η 13.5.5	1	1	68°53	294°00	68°50	194°06
ε 713	2	2	69°07	265°55	69°00	176°00

reflexion has been measured on the Goldschmidt goniometer and traced. In the following Goldschmidt's symbols are adopted.

In distinction from the datolites of the Andesite sopka the crystals of the Karagach deposit show a well developed prismatic zone and grow fast to the rock at one end of the Z-axis. The datolite crystals are of monoclinic system mostly with a well developed top d=2.93.

In the course of the measurements 37 forms have been detected. These forms are given in Table 2. In this table the indices and letter symbols are also given according to Goldschmidt and Dana.

The following forms are most developed: (100), (110), (120), (210), (111), (101), (011), (001), (522), ($\bar{5}22$), (342). The simple forms λ (111), ψ (104), W (211), U: (342), $\mathfrak{f}$ (522), α ($\bar{1}21$) are new for the Crimea; as to the forms p (141), ε (139), κ ($\bar{1}41$), λ ($\bar{4}73$), μ($\bar{7}43$), η($\bar{1}3.5.5$), ε($\bar{7}13$), they may be regarded as altogether new ones.

The face (011) is in many cases shaded, because of its repeated combination with (122), as it has been shown by S. P. Popov (²). Table 3 shows the results of measurements for new faces; as regards the faces known before, the results of their measurements closely approach those already published in literature and are therefore not given here.

As seen from the above accompanying figures (1, 2, 3, 4) face (001) varies as to size. The close joint occurrence of datolite with thompsonite produces the development of the faces (522), ($\bar{5}22$), (342) and a quantitative increase of the top faces (Fig. 1, 2). The pinacoid face (001) decreases. In crystals crystallized on calcite or the rock, the pinacoid (001) and the face (101) increase (Figs. 3, 4), while the face (011) decreases and the back part of the top develops the faces (211), ($\bar{1}11$) (Figs. 1, 2, 3, 4).

Of the new faces (141) and ($\bar{7}13$) have been encountered in two crystals. All the new faces with the exception of ($\bar{7}13$) are represented by small well-developed triangular planes and show good reflexions (Fig. 3).

The combinations of forms observed in the Karadag datolites are various. They are summarized by S. P. Popov, therefore we show here only the combinations of forms for the datolites from the Karagach range (Table 4).

Aside from the forms established for the Karadag the authors show also the following forms: h (340), Ω (410), s (103), l (031), Φ (1. 2. 12), y (241), β (142), χ (235), λ (321), B ($\bar{1}42$), Π_0 ($\bar{1}16$), P ($\bar{2}.2.15$), A_0 ($\bar{1}.1.12$). Thus, two types of crystals may be established for the Karadag. The chief difference between these crystals consists in the presence of a well-developed prismatic zone in the datolites of the Karagach range. Both from their forms and combi-

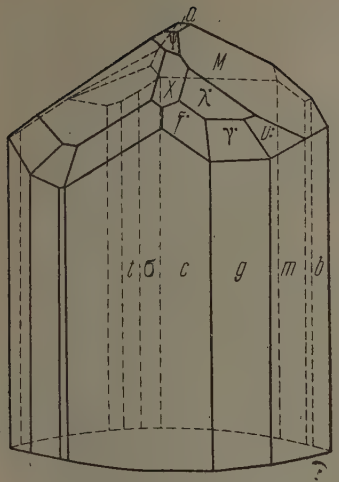


Fig. 1.

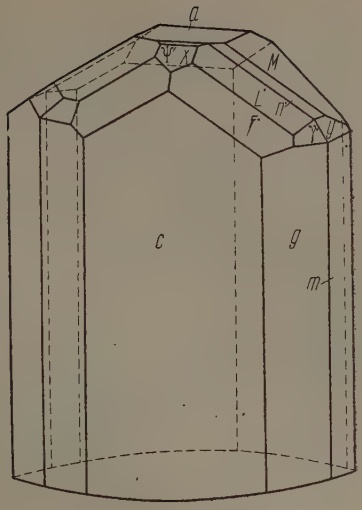


Fig. 2.

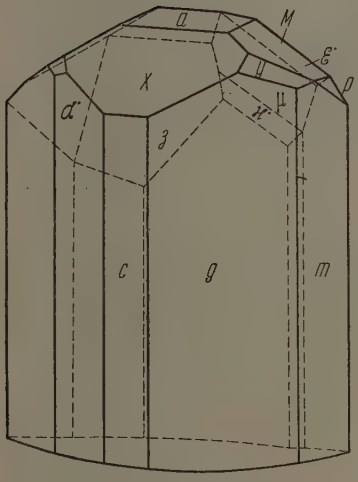


Fig. 3.

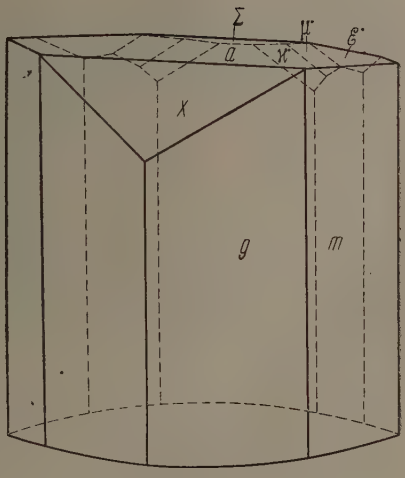


Fig. 4.

Table 4

No.	Combinations
1.	*a*, *b*, *c*, *m*, *t*, *g*, *σ*, *λ*, *x*, *n*, *ψ*, *o*, *M*, *ν*, *ξ*, *ɜ*, *i*, *κ*, *ʃ*, *U*, *н*, *m*,
2.	*a*, *c*, *m*, *g*, *σ*, *λ*, *x*, *n*, *ψ*, *W*, *U*, *L*, *ʃ*, *ξ*, *i*, *ν*, —,
3.	*a*, *c*, *m*, *g*, *x*, *M*, *Q*, *p*, *e*, *ν*, *ε*, *Σ*, *μ*, *α*, *U*, *ϰ*, *л*, *м*
4.	*a*, *c*, *m*, *g*, *x*, *ε*, *ξ*, *Σ*, *μ*, *ɜ*, *ω*, *∂*, *i*,

nations and from the mineral associations, the datolites from the Karagach range are closely resembling the datolites of the zeolite deposits of Lake superior described by Osann (7).

The crystals of the Andesite sopka possess a barrel-like habit and remind of the forms and combinations of the Tyrol datolites, described by Richelmann (6) and of the datolites from Bergen Hill in the USA.

As seen from the above data, datolite crystallizes after calcite and zeolites. This once more confirms the opinion of one of the present authors, expressed in a former article (4) that at the Karadag minerals are isolated in fissures and geodes in two phases. The present authors are inclined to believe that datolite was formed during the second phase, i. e. during the penetration of keratophyres, intersecting both andesites and tuffs. This assumption is confirmed also by the fact that the analcime encountered in the region of datolite findings is altered and (as shown by analyses) enriched in Ca and K, i. e. has been affected by emanations and keratophyres.

During the last years the investigators arrive at a conclusion that the zeolitization in basic effusive rocks is connected with the period of solidification of the post-volcanic process in the still non-solidified hot rocks. Consequently, the finding of datolite in the zeolite association and the fact that datolite crystallizes in some cases after thompsonite points to the hydrothermal isolation of datolite.

Boron in thermal solutions is contained in the form of anions $(B_4O_7)^{2-}$ or $(BSi_2O_6)^{1-}$. The presence of $(B_4O_7)^{2-}$ in thermal solutions is confirmed by the finding of boron in some thermals in rather large quantities. The presence of the $(BSi_2O_6)^{1-}$ anions in hydrothermal waters is based on the existence of the mineral sirlesite $Na{\cdot}BSi_2O_6{\cdot}H_2O$, precipitated in thermals and easily soluble in hot water without disintegration.

$(B_4O_7)^{2-}$ in the presence of $(SiO_3)^{2-}$ or $(BSi_2O_6)^{1-}$ with CaO, in the solution or from enclosing rocks, and forming datolite.

Received
9. V. 1939.

REFERENCES

[1] П. Мурзаев, Тр. Крымск. н.-иссл. ин-та, II, вып. 2 (1929). [2] С. П. Попов, Ежегодн. геол. минер. России, XII, вып. 5—6 (1911). [3] С. П. Попов, Минералогия Крыма (1938). [4] М. Шкабара, Учен. зап. Харьк. ун., № 6—7 (1936). [5] Görgey u. Goldschmidt, ZS. f. Krist., XLVIII (1911). [6] Riechelmann, ZS. f. Krist., **12**, 486; Hintze, Handb. d. Min., II, 173 (1897). [7] U. Osann, ZS. f. Krist., **24** (1865).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIV, № 2

GENETICS

INFLUENCE OF HOMOZYGOSITY AND HETEROZYGOSITY UPON QUANTITATIVE CHARACTERS

By P. F. ROKIZKY

(Communicated by N. I. Vavilov, Member of the Academy, 17. V. 1939)

The problem of heterozygosity is known to be of great importance both for theory and practice ([2], [3], [4]). Differences in the interpretation of the rôle played by heterozygosity in the development of the organism lead to different ideas on the ways and methods of breeding domestic animals and cultivated plants. In the majority of studies, many hundreds of them being available, heterozygosity or homozygosity is treated as a whole, i. e. many genes are studied at a time, and it is its cumulatif effect upon vitality, productivity and other features of the organism which is subjected to investigation. We decided to try and establish as precisely as possible the effect of an individual gene or, at least, of very few ones, in order to study their behaviour in homozygous or in heterozygous state. Such an approach is the more important that it should allow to find out to what extent are founded the assumptions of certain writers ([5], [6], [7]) as to the obligatory heterozygous action of genes, i. e. the more favourable effect of heterozygous state of the gene upon the development of the organism's properties.

The following 10 genes (9 loci) of the sex chromosome of *Drosophila melanogaster* were primarily used for the experiments: *y*, *pn*, *w*, w^e, *rb*, *v*, *m*, *f*, *B* and *Bx*. The following additional genes were included later on: *sc*, *ja*, *bi*, w^b, *cv*, *cm*, *ct*, *fw*, *sy*, *car*. Only part of these genes are included in the data presented below. This is due to the fact that, in order to study the influence of the genes in the heterozygote, it was necessary to reveal and to investigate the influence of the genes in question upon various characters. A special technique was applied to this; although simple, it required a work which lasted over 3 years. It consisted in crossing during many generations females heterozygous for some genes with males of a highly inbred and selected normal line (Caucasus XII) in which inbreeding was continued. By means of such a breeding system, the chromosomes and genes of the original mutant strains evidently were gradually replaced by chromosomes and genes of the inbred line, except the locus with the given gene and small regions to the left and to the right of it. Previously 10—12 generations were considered to be sufficient for isolating an individual locus and replace the rest of the genotype. After the mathematical studies by Haldane ([1]) it became clear, however, that this requires a very great number of generations. We had, therefore, to cross heterozygous females with males from inbred lines during many tens of generations (60—70 for certain genes). From time to time (usually every 7—8 generations) two cate-

gories of males which appeared, viz., normal ones and those possessing the mutant gene in question, were compared with each other for various qualitative and quantitative characters, namely, for the number of thoracic and abdominal (on the one of the segments of the abdomen) bristles, length and width of the wing, width of the head, length of the thorax. Measurements were expressed in divisions of the ocular micrometer.

It appeared in single cases that the difference between the two categories of males was steadily maintained during a large number (30—40) of generations; this made it possible to establish the influence of a very limited chromosome section bearing the gene in question (or of the very gene) upon some or other character and later to compare its influence in homozygous state with that in heterozygous state (in females). The study of the effect of genes upon various characters is of interest by itself. Data were secured bearing upon this question which will be reported in another communication. Here will be considered only the results of a comparative analysis of the genic action in homo- and heterozygous state upon the character which served to discover the influence of the given gene. Aside from this on the example of some of the genes was studied the influence of the homozygous and heterozygous state upon the biological properties as shown by the «output», i. e. the number of offspring obtained from one female within a definite number of days during which eggs were laid. This index embraces both fertility of the females and viability of the offspring.

As in each vial only 2 categories were bred out of the 3 which had to be compared, one had to compare singular categories from various types of crosses made parallel; this comparison gave in the great majority of cases a coincidence which was sufficient for current conclusions. Data on all the genes studied are summarized for the sake of convenience in one table (in which parallel control groups are not included).

Results of a Comparison between the Homozygous and Heterozygous Females for All the Genes and Characters Studied

Genes and number of the generation studied		F^{w}_{45}	F^{m}_{45}	F^{m}_{45}	F^{m}_{45}	F^{m}_{53}	F^{pn}_{50}	F^{w}_{56}	F^{m}_{60}	F^{y}_{56}	$F^{w^b}_{16}$	F^{cv}_{32}
Character		Number of thoracic bristles	Length of the wing	Width of the wing	Length of the thorax	Width of the head	Length of the wing	«Output» of the offspring	«Output» of the offspring	«Output» of the offspring	«Output» of the offspring	«Output» of the offspring
M of the corresponding categories of females	$\frac{a}{a}$	19.19	24.87	16.24	25.27	11.02	43.40	127.4	—	110.7	123.8	73.0
	$\frac{+}{a}$	19.49	39.54	27.29	26.21	11.04	43.41	122.8	95.4	106.9	118.4	112.3
	$\frac{+}{+}$	20.42	40.45	28.27	26.46	11.81	43.85	117.0	101.7	102.4	111.0	116.3
Number of individual studied . .		733	425	411	452	192	256	55 068	22 901	37 643	30 055	9 786
Number of cultures		—	—	—	—	—	—	460*	231	361	206	94
Per cent of dominance		76	6	8	21	ab. 100	ab. 100	56	—	56	58	9

* In the analysis of the «output» each culture corresponds to one female (homozygous or heterozygous) studied for the number of offspring.

In the case of quantitative characters either a nearly complete dominance was observed, or a very weak one, approaching «recessiveness». It is of interest that the gene miniature which is usually held for recessive is, as a matter of fact, not completely recessive with regard to its effect upon the wing. Unfortunately, it is impossible to decide whether the influence upon the length of the thorax belongs to the same gene or to some other which is closely linked with it. An intermediate degree of dominance is as a rule observed for the «output» of flies; it approximates 50 except for crossveinless. In the analysis for this character heterozygous females segregated for males and partly for females, too, when crossed with mutant males. The question arises in this connexion whether or not artificial decrease in the number of offspring from the heterozygous females took place owing to decrease of one of the categories competing with individuals of the other. A special analysis of the number of offspring of the two sets in each of the categories of crosses has shown that heterozygotes were correctly found to be intermediate, although in some cases there was a slight tendency toward the increase in the number of normal sons as compared to mutant ones. Evidently further analysis of the problem as to the rôle of struggle for existence between various groups of offspring is needed to judge of their average viability.

As to the position assumed by heterozygotes with reference to homozygous forms. a similar aspect is observed irrespective of the degree of dominance, namely, heterozygous forms develop a character which is more or less intermediate between the two types of homozygotes, in other words, they exhibit neither plus, nor minus heterosis. Besides, it makes no difference in which of the two homozygous forms, the mutant or the normal one, the character is stronger expressed. Thus, the «output» of offspring in the homozygotes $\frac{y}{y}$, $\frac{w}{w}$ and $\frac{w_b}{w_b}$ is higher than in the corresponding homozygotes $\frac{+}{+}$. In $\frac{cv}{cv}$ a reverse situation is observed, etc.

In no case did we observe the phenomenon of obligatory heterozygosity. We believe that instances of obligatory heterozygosity which should be naturally scrupulously checked in each separate case, can be but isolated ones and ought not to be used to account for the effect of inbreeding and heterosis. This does not cancel the question on that selection may, under definite conditions, be of benefit to heterozygotes as compared to some or other homozygote; this should take place, for instance, when selection proceeds in the direction of the average value of the character, not the extreme one, or when heterozygotes possess a different type of variability.

Moscow Zootechnical
Institute.

Received
9. V. 1939.

REFERENCES

[1] Bartlet a. Haldane, Journ. Genet. **31**, No. 3 (1936). [2] R. A. Fisher The genetical theory of natural selection, Oxford (1930). [3] J. L. Lush, Animal breeding plans, Jowa (1937). [4] S. Wright, Genetics, **6**, No. 2 (1921). [5] Альтшулер, Борисенко, Поляков, Биол. журнал, IV, № 3 (1935). [6] Борисенко, Альтшулер, Поляков, Биол. журнал, IV, № 4 (1935). [7] Д. А. Кисловский, Изв. Акад. Наук СССР (1937).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIV, № 2

GENETICS

INTERSPECIFIC HYBRIDIZATION IN THE GENUS *TARAXACUM*

By V. A. KOROLEVA

(Communicated by N. J. Vavilov, Member of the Academy, 28. IV. 1939)

The genus *Taraxacum* among whose representatives the Soviet workers ([8]) have discovered rubber-yielding species (*Taraxacum kok-saghyz* Rod. and *T. hybernum* Stev.) is of great interest in connexion with the fact that asexual reproduction (parthenogenesis) is common in it. Just a few species of this genus ([4,7]) are normal in respect to sex: the most promising soviet rubber-plant, *T. kok-saghyz* belongs to this group. Parthenogenesis greatly hinders classification within the genus *Taraxacum*; the number of species distinguished by various authors depends, therefore, on personal judgment. Thus, Handel-Mazzetti ([5]), the author of the first monograph on this genus, adopted such a broad concept of a species that sometimes was himself in difficulty while delimiting species. In his monograph are described 57 species. The next to publish a monograph on this species was Dahlstedt ([1,2,3]) who during many years was studying species of *Taraxacum* (mostly Swedish one); he has adopted very narrow limits so that *T. officinale*, for instance, is given by him a rank of a section with many species (section *Vulgaria* Dahlst.). Many writers think that various kinds of asexual propagation, parthenogenesis included, are the immediate result of hybridization [Ernst, Holmgren, Winge, Poddubnaja-Arnoldi ([7])]. So far I am aware, nobody has either described or experimentally produced hybrids between species of *Taraxacum*.

In my studies on *Taraxacum* containing no rubber and growing as weeds in plantations of kok-saghyz *, I have found a number of plants which proved to be sterile. These plants were morphologically intermediate between *T. kok-saghyz* and *T. multiscaposum* Schischk. Sterility of the plants discovered made me suggest their hybrid origin. In order to check this late in 1935 I have made crosses *T. kok-saghyz*×*T. multiscaposum* and the reverse. All crosses gave seeds; they were formed, however, not more than in 50% of the ovaries. On the average 45% of the seeds from the direct cross germinated, the individual variation being from 0 to 100%; the reciprocal cross yielded seeds with 16% germination which varied from 0 to 75%. Despite the fact that crossings were made without castration all the surviving plants proved to be hybrids of intermediate type. The easiness of obtaining hybrids between the species in question speaks for their good crossability; however, the hybrids proved to be highly sterile. Toward the time of the first autumn frosts in 58.3% of

* Culture was grown of original seeds from the Kegensky region of the Kazakh Socialist Soviet Republic.

cases seeds were found in the open pollinated heads of the hybrids; however, these were not more than 18.6% of normal seeds and germination was very low, on the average being equal to 6%. In 1937 the hybrids produced no seeds at all. The causes of sterility of the hybrids were studied by Poddubnaja-Arnoldi who worked in contact with us. Below follows a short characteristic of the two species in question and of their hybrid. The characteristic features of *T. kok-saghyz* are: absence of grey coating on the involucral bracts which are adherent to the inflorescence, presence of specific horn-like outgrowths on the involucral bracts, solid consistence of the somewhat greyish leaves which vary in the character of incision but lack acute indentations. *T. multiscaposum* is characterized by the grey coating and absence of horn-like outgrowths on the involucral bracts which are curly at the base; the leaves are of a less dense consistency, of darker colour and have acute indentations. Hybrids between these two species possess involucral bracts more or less deflected or even curled at the base of the inflorescence, always possess horn-like outgrowths but of another sort than those in *T. kok-saghyz*; they possess grey coating and leaves which approach *T. multiscaposum* in consistency, coloration, type of indentation. With regard to rubber-content the hybrids are inferior; on the average they contain 0.81% ranging from traces to 7.88%. Seeds produced by open pollination gave plants of two kinds, viz. identical with or closely similar to *T. kok-saghyz*, and plants approximating *T. multiscaposum* but resembling F_1 hybrids.

Cross-fertility of these plants was in the first case 55%, in the second 8.3%; plants of the *T. kok-saghyz* type were either fully fertile or entirely sterile whereas fertile plants of the type of *T. multiscaposum* produced not over 2—3% of seeds which besides failed to germinate almost completely. The most interesting is that as a result of open pollination of the hybrids, plants arose which did not differ externally from *T. kok-saghyz* but proved to contain only traces of rubber; this was undoubtedly due to hybridization with a species containing no rubber. Owing to the fact that the hybrids *T. kok-saghyz* × × *T. multiscaposum* were self-sterile and nearly cross-sterile, we have pollinated with their pollen plants of *T. kok-saghyz*. No emasculation was done here either. As pollen and embryosacks are greatly sterile in these hybrids (as shown by Poddubnaja-Arnoldi), the per cent of normal seeds did not exceed 24 and the average germination per cent was 21.8. As a result, plants of a similar type were obtained as in case of open pollination, i. e. 50% toward *T. kok-saghyz* and 50% toward *T. multiscaposum*; the combinations of the parental characters, moreover, were very interesting. In this case plants were also obtained which did not differ externally from *T. kok-saghyz* but contained only traces of rubber. Cross-fertility of the plants approaching *T. kok-saghyz* was on the average 85%, whereas plants of the other type had only 20.5% fertility; in the latter, however, the percentage of normal seeds did not exceed 2-3%. Finally, from pollination of *T. kok-saghyz* with pollen of a F_2 hybrid obtained from a cross *T. kok-saghyz* × F_1 hybrid (*T. kok-saghyz* × × *T. multiscaposum*) we obtained among 22 plants only one of the type of the hybrid *T. kok-saghyz* × *T. multiscaposum*; the remaining 21 plants were typical *T. kok-saghyz* and contained on the average 5.22% of rubber. Their cross-fertility was, however, 89.5% and seeds were either formed or not formed at all. It is interesting to note that from pollination of *T. kok-saghyz* with pollen of F_2 hybrid of the *T. kok-saghyz* type all the plants were of the *T. kok-saghyz* type, but their rubber content varied from traces to 6.33% and fertility was reduced (95.7%). In none of the interspecific hybrids in the genus *Taraxacum*, as pointed out by Poddubnaja-Arnoldi, parthenogenesis was found. Crossing *T. kok-saghyz* with another sexually normal species *T. karatavicum* Pavl. presented the same aspect as crossing with *T. multiscaposum*, i. e.

F_1 was intermediate and contained an amount of rubber varying from traces to 4.53%. Seedlings of the hybrids of *T. kok-saghyz* with another sexually normal species *T. bessarabicum* H. M., died at early age so that I could not obtain any adult hybrids. Aside from these crosses I have crossed *T. kok-saghyz* with the following parthonogenetic species: *T. brevicorniculatum* Korol., *T. microspermum* Schischk., *T. officinale* Wig., *T. Hoppeanum* H. M., *T. tianschanicum* Schischk., *T. hybernum* Stev. In spite of the great number of crossings no hybrids were obtained.

Summing up, we arrive at the following conclusions:

1. Parthenogenesis can hardly be considered to be a result (so much the more an immediate one) of hybridization.

2. Hybridization of *T. kok-saghyz* with *T. multiscaposum* (and the reciprocal) occurs both in natural populations of *T. kok-saghyz* in Tian-Schan and in plantations or in experiment.

3. Pollination of *T. kok-saghyz* with pollen of the hybrids *T. kok-saghyz*×*T. mulatiscaposum* produces plants which differ in nothing from *T. kok-saghyz*, but contain just traces of rubber; hybrids *T. kok-saghyz*× ×*T. multiscaposum* should be therefore, carefully removed from plantations of *T. kok-saghyz*.

4. Hybrids with parthenogenetic species might be of great interest for breeding purposes, since these species possess a number of valuable characters which are absent in *T. kok-saghyz*, viz., rapid development, vigour, thickness of the root, great productivity, finally, parthenogenesis. However, crossing with these species gave no positive results. The reason for this was discovered by Poddubnaja-Arnoldi. Crossing with species normal in respect to sex (*T. multiscaposum*, *T. karatavicum*) occurs readily, but is of no interest, the hybrids being sterile and possessing no practical value.

All-Union Institute of Plant Industry.
Leningrad.

Received
16. V. 1939.

REFERENCES

[1] Dahlstedt, Acta Fl. Sveciae, I (1921). [2] Dahlstedt, Kungl. Sv. Vetensk. Handl. Tredje, Ser. V, 3 (1928). [3] Dahlstedt, Acta Horti Bergiani, IX (1929). [4] A. Gustafson, Hereditas, **16**, № 1—2 (1932). [5] Handel-Mazzetti, Wien (1907). [6] Longberg, Acta Bot. Fennica, **17** (1935). [7] Поддубная-Арнольди и Дианова, Бот. журн. СССР, **22**, № 8 (1937). [8] Rodin, Acta Inst. Bot. Acad. Sc. USSR, Ser. I, Fasc. I (1934).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXI, № 2

GENETICS

RESULTS OF AN ANALYSIS OF SEGREGATION OF TOMATO HYBRIDS CONDUCTED FOR EACH FAMILY SEPARATELY

By T. K. ENIN

(Communicated by N. I. Vavilov, Member of the Academy, 13. V. 1939)

Data on segregation in F_2 are given in the genetical and breeding literature mostly in the form of sum total for all the progenies derived from the F_1-plants, and not individually for each family. Such an analysis gives no idea as to the character of the F_2-segregation in individual families. Indeed, those critically minded towards Mendelism profited by the occasion not only to throw doubt upon the existence of any regularity in the segregation of hybrid families, but even to question the very fact of segregation. It was on account of a number of experimental works and critiques directed against Mendelism ([1–4]) that we have partly repeated the experiments made by Mendel (1861), and Bateson (1905) on peas, using our combinations of pea-crosses; the experiments have shown all the F_2-families to segregate following Mendel's sheme ([5]). In the present paper are contained the results of an analysis for the leaf shape in individual families of tomato hybrids, this plant being more convenient as compared to peas. Tomato are known to be characterized by a high coefficient of propagation; the analysis of the F_2-segregation for leaf shape requires neither important outlays, nor labour. The leaf shape in tomato is, besides, quite a definite character, liable to no sharp variation under the influence of external factors, and thus having advantage of such characters as shape and coloration of seeds in peas. Mendel himself warned against the variability of these characters, but, unfortunately, the confusion still persists there.

The tomato-cross which served as a material for this work was made by M. I. Hadjinov (Leningrad). Variety «Mikado» served in this cross as the pistil plant; it is characterized by leaves resembling those in potato. As the pollen parent was taken the variety «Erliana» which differs from «Mikado» in having normally dissected leaves and in other characters as well. Emasculated flowers of «Mikado» were pollinated with a mixture of pollen taken from different plants of «Erliana». The tomato plants were cultivated following the commonly accepted scheme, i. e. seeds were first planted into boxes (in the greenhouse), seedlings were grown under application of pinching of young plants, and, finally, planted into the open. Following the description all the F_1-plants were characterized by normally dissected leaves. For obtaining F_2, from 2 to 4 first-to-mature fruits were taken from each F_1-plant. The remaining fruits failed to mature and were therefore not taken for seed. In order to enhance the analysis of segregation, the seeds taken out of each individual F_1-fruit were planted in separate boxes. The analysis of the F_2-segregation for leaf shape was made on the 15th—30th day after planting seeds (Fig. 1). The development of the young plants was stimulated by the favourable conditions of the greenhouse. It should be noted that the plants from earlier planting (February 21) developed much worse than those from seeds

planted later (April 3); in the latter group death of the young zygotes was almost nil. The plants sown first suffered of lack of light and heat, owing to which fact many of them died at the time of germination.

Fruits of 50 well developed F_1-plants (out of 100 available) were collected for the work.

To obtain F_2-seeds of 25 F_1-plants only were planted. Segregation analysis carried out for individual families is presented in the table.

F_2-Segregation from the Tomato-Crossing «Mikado» × «Erliana» Presented by Individual Families

Plant number F_1	Family number F_2	Number of plants obtained in family F_2	Leaf shape observed		Leaf shape expected		Error $\pm m$	Note
			Normally dissected	Potato-like	Normally dissected	Potato-like		
5	1	365	270	95	273.75	91.25	8.2	
6	2	351	283	68	263.25	87.75	8.1	
7	3	100	78	22	75.0	25.0	4.3	
8	4	497	381	116	372.75	124.25	9.6	
14	5	218	164	54	163.15	54.5	6.3	1st term of planting in the greenhouse
16	6	259	195	64	194.25	64.75	6.8	
17	7	143	104	39	107.25	35.75	5.3	
18	8	319	252	67	239.25	79.75	7.7	
21	9	211	161	60	158.25	52.75	6.2	
32	10	215	171	44	161.25	53.75	6.2	
2	11	482	365	117	361.5	120.5	9.5	
11	12	471	363	108	353.25	117.75	9.4	
12	13	395	294	101	296.25	98.75	8.4	
13	14	257	196	61	192.75	64.25	6.8	
19	15	692	518	174	519.0	173.0	12.0	
22	16	534	400	134	400.5	133.5	9.8	
23	17	595	448	147	446.25	148.75	10.5	2nd term of planting in the green house
26	18	266	205	61	199.5	66.5	6.9	
27	19	694	519	175	520.5	173.5	12.1	
34	20	447	330	117	335.25	111.75	9.1	
35	21	353	265	83	264.75	88.25	8.1	
36	22	234	175	59	175.5	58.5	6.6	
38	23	438	333	105	328.5	109.5	9.0	
39	24	219	166	53	164.25	54.75	6.3	
40	25	416	315	101	312.0	104.0	8.6	
Total:	25	9171	6951	2220	6878	2293	40.1	

From the table it may be seen that in all cases without any exception segregation in the F_2-families occurred according to Mendel's scheme (3 : 1). This is in full accord with the literary data on tomato genetics ([6]). In single F_2-families there were deviations from the theoretical expectation, which is accounted for by the death of zygotes with normally dissected or potato-like leaves as a result of unfavourable external conditions.

Institute of Genetics.
Academy of Sciences of the USSR.

Received
17. V. 1939.

REFERENCES

[1] Т. Д. Лысенко, Яровизация, № 1—2 (1938). [2] Н. И. Ермолаева, ibid., № 1—2 (1938). [3] М. М. Аранчук, ibid., № 4—5 (1938). [4] П. И. Титов, Социал. зернов. хоз., № 5 (1939). [5] Т. К. Енин, Докл. ВАСХНИЛ, № 6 (1939). [6] J. W. Mac Arthur, Genetics, **11** (1926). [7] J. W. Mac Arthur, ibid., **13** (1928). [8] J. W. MacArthur, Trans. Roy. Canad. Inst., **18** (1931).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIV, № 2

GENETICS

EVOLUTION OF CULTIVATED RASPBERRY

By M. A. ROSANOVA

(Communicated by N. I. Vavilov, Member of the Academy, 16. V. 1939)

The first reference to raspberry as a cultivated plant was made in the third century before our era; the first data on its varieties appeared at the end of the XVI century, while the beginning of breeding and a more definite varietal differentiation can be traced as late as by the beginning of the XIX century. For the history of cultivation of raspberry see Card (6), Bunyard (5) and Hedrick (9). To the same epoch should be referred the appearance of certain varieties still cultivated (as Red Antverpen, White Antverpen) whose fruits materially differ in size from wild-growing raspberry.

More extensive work on cultivated raspberry is thus not more than 150 years old; one may, therefore, assume that man has just lately started to transform this plant to suit his demands.

The young age of the culture in question is by itself of great interest as the ways of evolution can be better followed through a short period of time. To find out changes that occurred during this short time one should establish the principal features which distinguish cultivated forms of raspberry from the wild ones.

Cultivated varieties of red and white raspberry (aside from dewberry-like black and purple ones) belong to two species: *Rubus vulgatus* Arrhen., common European raspberry, and *R. strigosus* Mchx., American setose raspberry, as well as to hybrids between these species.

Rubus vulgatus, *R. strigosus* and the Siberian raspberry *R. sachalinensis* Léveillé all belong to the conspecies *R. idaeus* L. (3); *R. sachalinensis* did not take part in the formation of varieties.

R. vulgatus, European raspberry, is distributed in a wild state over the whole of Europe and in Western Siberia; it is confined mainly to the zone of woods. The northern border passes near the Polar Circle, in the south raspberry is found in the mountains of Caucasus, of Asia Minor and Middle Asia.

R. strigosus, American raspberry, is distributed over the north-eastern part of America, in the mountains of Georgia to Pensylvania and from Main to Dakota; it occurs in Canada and in the west, in British Columbia.

As to the geographic distribution of the cultivated raspberry, the varieties which belong to *R. vulgatus* and to the hybrids *R. vulgatus* × *R. strigosus* are distributed over Europe, Western and Eastern Siberia and in North America; varieties which belong to *R. strigosus* in North America are as common as those belonging to *R. vulgatus* and are less common in Europe.

As a result of a study on variation in the whole collective species *Rubus idaeus* in nature and under cultural conditions, conducted on 90 samples of wild-growing strains collected at various places in Europe, Asia and America, as well as from a study of varieties from various countries (232 varietal samples), we could establish that the following changes occurred in raspberry under cultivation.

1. The mean weight of fruits varies from 0.4 to 1.5 g in the various wild strains; it is significantly shifted toward greater weight and varies in different varieties from 1.0 to 3.0 g.

2. Owing to interspecific hybridization between European and American raspberry, as well as to extensive interspecific hybridization in many cases between distants trains new combinations of characters appeared and new characters not peculiar to wild-growing strains, the range of variation growing more extensive. Thus, for example, in varieties belonging to European raspberry, there occur fruits with an intake which is unknown in the wild strains; the polymorphism of the fruits is much stronger pronounced in cultivated raspberry than in the wild one.

3. In the cultivated raspberry varieties are more frequent which are characterized by recessive characters (yellow fruits, spineless or poor in spines, remontant varieties); wild-growing strains with recessive characters are exceedingly rare in nature.

4. Great caryological variability arises in cultivated raspberry. While all the wild strains belonging to *R. vulgatus* and *R. strigosus* always possess 14 chromosomes, cultivated raspberry belonging to *R. vulgatus* or to hybrids *R. vulgatus* × *R. strigosus* often includes varieties with 21 and 28 chromosomes ([1],[2],[7],[8]). This phenomenon is evidently connected with hybridization processes which in cultivated raspberry proceed more intensely. It should be also noted that large fruit size in cultivated raspberry is not correlated with polyploids. Polyploid varieties commonly have fruits of moderate size (mean weight 1—2 g). On the other hand, large fruit size is undoubtedly connected with hybridization and with heterosis which is fixed by vegetative propagation.

5. Cultivated raspberry is in the majority of cases self-fertile, whereas wild strains are, as a rule, self-sterile ([4]). It is thus obvious that hybridization and selection practised by man played a great part in the evolution of cultivated raspberry.

It is possible that mutation processes are enhanced in connexion with hybridization; owing to this new characters are produced more often and those of them which prove useful are fixed in cultivation.

Hybrid varieties formed and especially polyploid ones acquire a new ecological adaptability and, therefore, invade areas which may be suitable to a distinct eco-morphological type.

Owing to the recent origin of the majority of hybrid varieties and to the extensive primary variety-test cultivated raspberry passed through, no ecological groups of varieties are yet distinguishable.

When one compares the ecological differentiation of the wild-growing strains within the whole conspecies R. *idaeus*, one should see that climatic differentiation of varieties does not correspond at present to climatic differentiation of the species. Thus, Siberian wild raspberry, *R. sachalinensis*, differs in many characters from the cultivated varieties grown in Siberia and belonging to European raspberry, or to hybrids between European and American raspberries; the wild-growing North-American strains also differ in their characters from the cultivated European varieties which amount there up to about 50%.

Close coincidence between the ecological grouping of the wild-growing and

cultivated strains is observed in Europe where cultivation of raspberry has begun at earliest date and where man has already selected the most adopted types.

On the basis of our data the following two climatypes may be traced among wild-growing strains of European raspberry: 1) northern and north-eastern European climatype which is mostly characterized by a spreading habit of growth, by earlier blooming and by frost resistance; 2) southern climatype which is characterized by half-spreading or erect habit, by broader leaves, by somewhat later blooming and in some cases by a remontant habit.

The European varieties may be, according to these two climatypes, divided into northern ones which are earlier, frost resistant and have a spread habit (like Usanka, Seedling of Spirin, Novost of Kuzmin, Slava Severa) and southern remontant ones which have a half-spreading habit (like remontant Billiard, Seedling of Baumford).

As already stated within cultivated raspberry these ecological groups are insufficiently distinct, so that one may point out discrepancies between inconsistency with a definite ecological type (for example, variety Malborough which is distributed in the north has erect habit; the remontant variety English yielding good results in the south has a spreading habit).

Cultivated raspberry is thus passing its initial stage of ecological differentiation. Man selects from the natural population desirable forms, recessives included; he carries on a great work of creation of new forms by means of artificial hybridization or by means of selection among seedlings obtained from spontaneous hybridization and, in continuing this work, subjects to extensive trials the varieties created by him. The latter circumstance should lead in the future to a more clear-cut ecological differentiation of the cultivated raspberry; at the present it may be traced only among European varieties belonging to *Rubus vulgatus*.

Received
17. V. 1939.

REFERENCES

[1] В. Петров, Тр. Центр. ген. плодо-ягод. лаб. им. Мичурина (1934). [2] М. Розанова, Ягодоведение и ягодоводство (1935 и 1937). [3] M. Rosanova, C. R. Acad. Sci. URSS, XXIV, № 1 (1939). [4] Н. Чуксанова, Садоводство (1939). [5] E. J. Bunyard, Pomol., **3**, No. 1 (1922). [6] F. Card, Bush fruits (1919 a. 1925). [7] N. Chomisury, Angew. Bot., **9**, No. 6 (1927). [8] G. Darrow a. A. Longley, J. of Agr. Res., **27** (1924). [9] U. Hedrick, The small fruits of New York (1925).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIV, № 2

GENETICS

PHYSIOLOGICAL DIFFERENCES IN DRAFT-HORSES AS CORRELATED WITH THEIR PERFORMANCE

By V. I. PATRUSHEV

(Communicated by N. I. Vavilov, Member of the Academy, 14. V. 1939)

In the course of our work on methods for evaluating the various breeds of farm animals by physiological factors in respect of horses we set ourselves the aim of solving the following two questions: 1) whether and to what extent it is possible to evaluate the quality of these animals on the basis of our investigations; 2) whether and to what extent it is possible by means of physiological and biochemical investigations to determine the work performed by a horse with regard to its greater or lower arduousness.

Our investigations were made at the Moscow Hippodrome during the 1938 season. The material consisted of 13 Ardennes and Brabançons, two years old; they were investigated by us before and after training as well as before and after work and also when resting after tests for speed and maximum load hauls. To the study were subjected the respiratory, cardio-vascular and circulatory systems, as well as some of the reduced oxy-fermentation processes. The strain on the organism exerted by the various tests is evident from the diagram showing the relative physiological changes during work (Fig. 1).

During the tests for maximum load haul the animals showed the greatest number of pulsations, the greatest relative volume of blood cells and dry residue of the blood. These tests show also the greatest increase in the total and reduced glutathione of the blood and, vice versa, the least decrease in the oxidized glutathione as well as of the content of total glutathione in a single erythrocyte. In addition, during these tests the Brabançons show a greater increase of lactic acid than during the other tests. The Ardennes come second for increase of lactic acid during load hauling tests. In the case of Brabançons respiration is more frequent in tests for speed (running at a trot) than in tests for maximum load hauls; in the case of Ardennes respiration is more frequent in tests for speed with load than in tests for maximum load hauls. Retardation in the sedimentation of erythrocytes for 30 min. (and even 60 min. in the case of the Ardennes) after the test is least significant as compared with such retardation in the other tests. Thus, on the basis of the functional changes in the fundamental systems of the organism, the tests for maximum load hauls may be concidered the most arduous for a horse.

The tests for a 10-km haul of load at a steady pace and the tests for a 5-km run at a trot differ less from each other in their effect on the horses. The latter tests evidently exert a somewhat greater strain on the horses than the former. After a run at a trot the Brabançons show a maximum increase in frequency of respiration (more frequent than during tests for maximum load hauls). Regarding rate pulse during tests for speed at a trot Ardennes take second and Brabançons third place. For increase in the relative volume of blood cells

and dry residue of blood the Ardennes take likewise second place. The Ardennes and Brabançons hold first and second place respectively for increase in lactic acid and after tests for a run at a trot. We believe that tests for maximum load hauls are harder for horses than tests for a 5-km run at a trot in their turn, the latter tests are harder than the tests for a 10-km load haul.

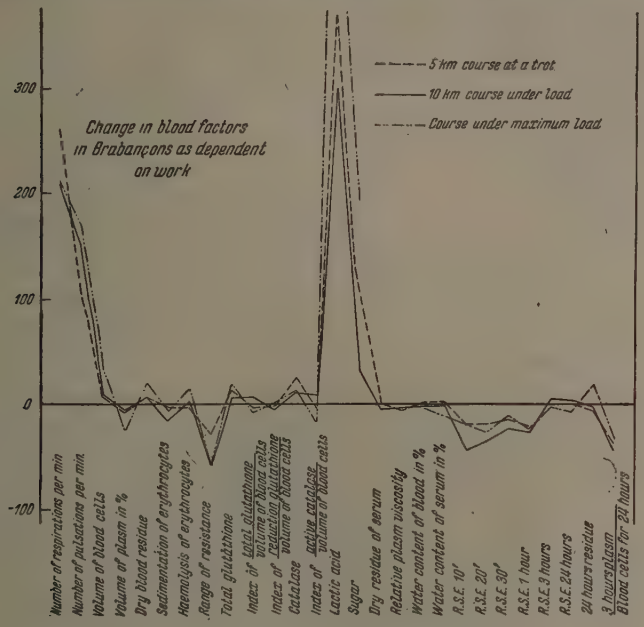


Fig. 1.

Thus, on the basis of such investigations it is possible to evaluate the arduous nature of work by the strain it imposes on the animal's organism.

At all the tests the horses had to show the maximum performance. Therefore, the horses could be judged only on the basis of maximum expressions. As to the expediency of the changes in the various factors as well as the coefficient of efficiency, one could only draw an indirect inference from the data of the tests by comparing the places occupied by the animals with the absolute and relative indices of the functional changes. In the case of horses which do the same kind of work and show the least functional changes additional tests have to be introduced.

From our investigations we may draw the general conclusion that the best horses have the lowest rate of pulse when at rest. Notwithstanding their more arduous work, a number of winners, after the test, show a lesser increase in frequency of pulse than the non-winners. When employed on particularly arduous work, the best horses show a greater frequency of beats than the average and inferior horses. The variations in the frequency of respiration are similar to those of pulse.

Not all the best horses show the greatest concentration of red blood cells. Some of the inferior horses show high blood values. Among the Brabançons, Clan which occupied the last place at the tests, showed the highest volume of blood cells. The same horse showed the greatest frequency of pulse when at rest and nearly the greatest frequency after each of the tests. Thus, it

has a less trained and constitutionally weaker heart than the others. The high index of red blood cells in this case may be the result of compensation.

Increase in the number of red blood cells after work is not always correlated with best results of tests. The champion horse Svet whose cardio-respiratory system was least strained during the test for load haul at a steady pace showed the highest increase in the number of red blood cells. Another winner Linbruk, showed the greatest frequency of pulse and respiration and after the tests had an almost minimum volume of blood cells. Changes in the blood concentration of Ardennes after work are shown in diagram 2 (in the legend to the diagram the horses are given in the order of place occupied by them after the test).

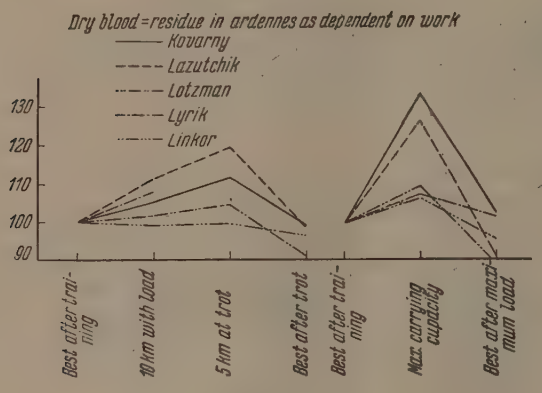


Fig. 2.

The best horses have a more intense blood formation and as compared with the inferior horses develop higher haemolythic processes after work. Allowance being made for concentration of erythrocytes, the best horses show a more rapid sedimentation of the erythrocytes. Retardation in the sedimentation of erythrocytes after training is more clearly expressed in the case of inferior horses. As compared with the non-winners, the winners show a more rapid sedimentation of the erythrocytes after each of the tests. However, even among the best horses there are some which show a great retardation in the sedimentation of the erythrocytes (Svet, Kovarny).

On the average the best horses after work show a smaller increase in lactic acid than the inferior horses, while the removal of the lactic acid proceeds at a much more rapid pace when the animal is at rest. Among the horses which were at the bottom of the list at the tests some showed an insignificant increase of lactic acid. On the other hand, some of the winners showed an increase in lactic acid approaching the maximum (Svet).

Hyperglycaemia is less strongly expressed in the best horses than in the inferior ones. Some of the horses which were defeated at the tests showed a considerable reduction in the sugar content of the blood below the normal while some showed the highest increase (nearly 340% above the normal).

The horses which did not win showed a highly increased catalase activity after the tests. The same was observed after the tests for speed with load.

Extreme absolute and relative shifts in total and reduced glutathione after work are typical of horses which are at the bottom of the list of the tests. Most of the best horses show medium increase. Inferior horses show a maximum reduction in the relative viscosity of blood serum after the tests.

We thus come to the conclusion that the complex physiological and biochemical investigations enables us to differentiate horses according to quality even within the limits of a homogeneous high class group, such as the well developed pure Ardennes and Brabançons, the offspring of the best sires and dams of the breed investigated by us.

Institute of Genetics.
Academy of Sciences of the USSR.

Received
14. V. 1939.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIV, № 2

MICROBIOLOGY

ANATOMICAL AND CYTOLOGICAL MODIFICATIONS IN CEREALS ATTACKED BY MOSAIC VIRUS

By V. ZAZHURILO and P. SCHWARTZ

(Communicated by N. I. Vavilov, Member of the Academy, 27. IV. 1939)

Mosaic diseases of cereals are rather widespread in the black-soil zone of this country. This has been established definitely by the investigation carried on within recent years at Voronezh Plant Protection Station (1). Since the infectiousness of these diseases has not yet been proved, a great interest belongs to every indirect datum bearing upon their nature.

To this kind of indirect data refer the anatomical and cytological modifications which are observed in plants attacked by virus diseases. Those which have been investigated by us were induced by the following types of cereal mosaic: rosette mosaic and green mosaic of winter wheat and winter rye, yellow mosaic of winter wheat, yellow mosaic of hard spring wheat, yellow mosaic of oats, green mosaic of oats, millet and maize, mosaic of couchg-rass.

The material for investigation was supplied chiefly by the Kamennostepnaja plant breeding station. Ordinary microscopical technique was resorted to in the tests. As fixations we used the 10% formalin, Karnoit liquid, Navashin's and Levitsky's fixations, the chromoacetic mixture and Regots' liquid. The parts investigated were largely the cross-sections of leaves from sound and diseased plants.

The most striking feature in the plants affected with mosaic was the non-homogeneous colouring of leaf's blade and vagina. On examining the mosaic portions it was established that where the colour was light-green or yellow the plastids were strongly underdeveloped; they were sparingly coloured, reduced in size and their number in a diseased cell was less than in a sound one. In the cells almost entirely deprived of plastids the contents present the nucleus with a granular mass of protoplasm sometimes gathered into a lump in one of the cell ends. The nucleus in the infected areas has usually an increased number of nucleoles (3—5 and more). Hypertrophy of the nuclei is of no seldom occurrence and is attended by modification of their outlines and vacuolization.

Deep derangement of the vascular system was observed in all the investigated types of mosaic cereals. In the phloem necroses are formed which are the more expressed, the sharper the mosaic pattern in leaves and the stronger the depression of plants. The necroses are especially strong in the rosettes of winter wheat and rye where they involve not only the largest part of the phloem, but the adjoining areas of parenchyma, as well.

In the leaf parenchyma they occur as separate, very typical foci. The necrosed cells in the latter are filled up with a substance strongly akin to a great many dyestuffs (haematoxylin, gentian-violet and others), which it will not release even on strong differentiation. The chloroplasts of those cells are highly vacuolated; when their number in the cell is great they touch each other to form a pattern of trabecular nets described by Dufrenoi. The reticulated picture receives added strength from the fact that the plastids stain more intensely in their inner layers lining the vacuoles than they do in the peripheral areas and than does the substance filling up the cell.

The necroses react positively to Millon's reagent, to suberin with gentian-violet and to wood with phloroglucin. The latter reaction points to the formation of wound gum, recorded by a number of authors in necroses of mosaic plants ([4], [6]). The necroses of the phloem are stained with sudan and this is also confirmed by literary data. All those facts lead us to the conclusion that the necroses observed belong to the group of sharp ones which are most typical for the virus diseases of plants.

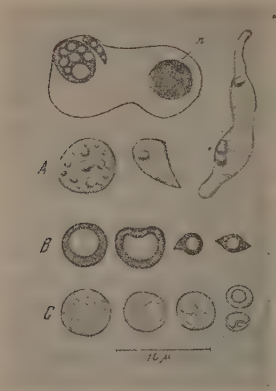


Fig. 1. Vacuolar bodies: *A*, rosette-mosaic (variety Harvest Queen); *B*, yellow mosaic of winter wheat; *C*, green mosaic of oats; *N*, nucleus.

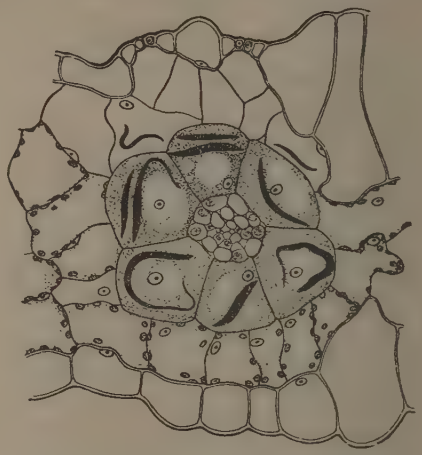


Fig. 2. Protein crystals in leaf parenchyma of green mosaic millet.

In nearly all types of mosaic cell inclusion bodies have been detected. They were especially numerous in green-mosaic oats, green and yellow-mosaic winter wheat, and in rosettes of winter wheat and rye. Contrary to this, their number was insignificant in yellow-mosaic oats, yellow-mosaic hard spring wheat and in mosaic leaves of couch-grass. And we failed to detect any such inclusions in millet and maize attacked by mosaic.

The inclusion bodies occur almost exclusively in light green portions. It is a difficult matter to detect them in normally stained areas, while in sound plants they fail to appear altogether. Their number grows as the disease makes its progress.

The cell inclusions observed were quite identical with vascular bodies studied by MacKiney and oth. ([8]) in wheat and Sukhov and Vovk ([2], [3]) in oats. In order to ascertain with more precision whether the inclusions observed might be identified with the vascular bodies occurring in oats affected with «Zakuklivanie», we investigated herbarium specimens of such oats from Western Siberia and Tuvin National Republic. The comparative study has revealed a complete similarity between the inclusions in various types of mosaic cereals and those in oats attacked by «Zakuklivanie».

Both in our material fixed with formalin and in herbarium specimens the cell inclusions presented colourless or slightly yellowish drops strongly refringent and provided with a thin coat. The coating becomes quite distinct after the slices have been kept in potassium bichromate or stained with gentian-violet. In formalin and herbarium materials the vacuolated bodies are exceptionally rare. Yet in materials treated with fixatives containing acetic and chromic acids the inclusions are usually vacuolated (Fig. 1, *A* and *B*). Strong vacuolation is observed if the slices have been kept in dissolved acids, say in acetic acid solution (Fig. 1, *B*), or stained with methylene green and aceto-carmin containing acetic acid.

The inclusion bodies stain well under the action of haematoxylin and gentian-violet. The coat and the inner layer enveloping the vacuoles stain more

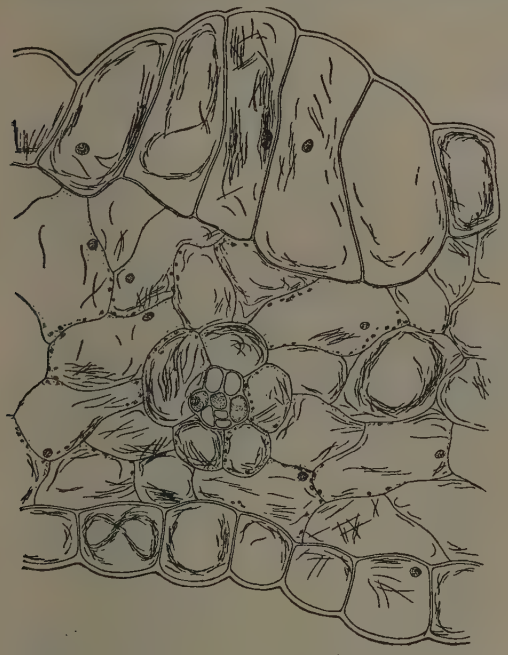


Fig. 3. Aciculate protein crystals in leaf cells of green mosaic maize.

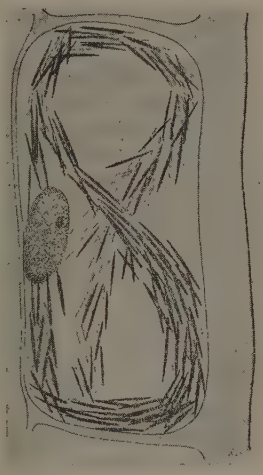


Fig. 4. A large eight-shaped crystal broken down into small aciculate ones. Epidermic cell of green mosaic maize.

intensely than do the body contents. The gentian-violet has proved an especially convenient reagent to detect the cell inclusions. The latter in mosaic oat plants are well stained with gentian-violet if the method used in detecting suberin (completed by differentiation with 5-10% hydrochloric acid) is resorted to. With this method applied the contents of the inclusions turn dark violet while the coat stains blue-green. And it is only the inclusion, the liquified and cutinized cell walls and the necrosed parts which retain colour after differentiation. In mosaic wheat and rye, however, the inclusion bodies refuse to stain on such treatment. With these crops good results are obtained when gentian -violet is applied according to Newton. The inclusion bodies are soluble in alcohol and chloroform.

Considering the results here reported we can subscribe to the opinion advanced by Rischkov and Sukhov ([5]) that the inclusion bodies are a product of pathological secretion of cells. They are extremely suggestive of the tanoid

bodies detected by Rischkov and Mikhailova in potato tubers, but the outward resemblance is not attended by identity of chemical responses and their reaction to tanning substances is negative. Different also is their behaviour in response to the dyestuffs. They refuse to stain when acted upon by safranin, fuchsin or the blue. Neither will they respond to the special tanoid body reaction carried on according to Pfeffer with methylene-blue. In oats, millet and maize affected with green mosaic and in rosettes of winter wheat protein crystals appear in the cells whose protein nature has been established by Millon's reaction. They dissolve in water and are stained with acid fuchsin and other dyes (haematoxylin, gentian-violet). As to their form, size, as well as genesis they are identical with the protein crystals, discovered by Sukhov in the epidermal leaf cells of oats, millet, maize and barley affected with mosaic.

While in the wheat rosettes the crystals were found only in the epiderm and in the oats and millet only in the mesophyll, they were observed in both the epiderm and mesophyll in maize (Fig. 2 and 3).

Conclusions

1. Mosaic diseases of cereals occurring in the Voronezh province are characterized by an array of anatomical and cytological modifications typical of plant (especially cereal) virus diseases (necrosed phloem, underdeveloped plastids, hypertrophy of nuclei attended with their vacuolization, increase in the number of nucleoles and formation of vacuolar bodies and protein crystals).

2. The presence of protein crystals in oats, millet and maize (Fig. 4) affected with green mosaic and in rosettes of winter wheat is a good ground to suppose that both the green and the rosette mosaic occurring in the Voronezh province are induced by the same virus as is responsible for the «Zakuklivanie» of cereals in Siberia and in north-eastern regions of the European part of this country.

Phytopathological Laboratory.
Voronezh Plant Protection Station.

Received
29. IV. 1939.

REFERENCES

[1] В. К. Зажурило и М. В. Горленко, Итоги научно-иссл. работ ВИЗР за 1936 г. (1937). [2] K. S. Soukhova. A. M. Vovk, C. R. Acad. Sci. URSS, XIX, No. 3 (1938). [3] K. S. Soukhova. A. M. Vovk, ibid., XX, No. 9 (1938). [4] П. В. Михайлова и Р. М. Пивоварова, Сб. Вирусн. бол. в Крыму и на Украине (1934). [5] В. Л. Рыжков, Вирусные болезни растений (1935). [6] K. Esau, Bot. Review, 4, No. 10 (1938). [7] A. Meyer, Analyse der Zelle (1920). [8] McKinney, Eckerson a. Webb, Journ. Agr. Res., 2 (1923).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIV, № 2

AGROBIOLOGY

A SIMPLE METHOD FOR DETERMINING THE INFECTION OF WHEAT SEEDS WITH *USTILAGO TRITICI*

By P. SCHWARTZ

(Communicated by N. I. Vavilov, Member of the Academy, 27. IV. 1939)

Wheat is infected with loose smut while it is growing. The mature spores wafted by the wind reach the ovaries to germinate there and to form mycelium. The latter penetrates the seed where it remains in quiescent state until the planted seed begins germination. Then it is awakened to growing with and within the seed. In field the infection with loose smut is usually estimated when the wheat comes into ear. But the field inspection fails to reveal properly how much had the seed material been infected. The degree of seed infection with loose smut cannot be inferred directly from the number of smut injured ears, because the meteorological conditions for infection during flowering as well as the susceptibility of the variety concerned must be taken into account. And there is no other reliable method for judging about the extent of infection of seeds except to detect the spawn in the seed material itself. The technique devised should be neither laborious nor time consuming in order to fit the purpose of large scale analysis.

The search for the method of testing seeds for their infection with loose smut has been the subject of several works published within recent years. Budrina (2) checked the anatomical method of analyzing wheat's seeds and seedlings for loose smut infection and found it suitable for working on a small scale.

In some or other form this method was applied by Bubentzov (1) in his attempt to devise a laboratory technique for ascertaining the condition of mycelium in the seed.

Examination of the available methods intended to reach the purpose was done by Shcheglov (5). His test involved: 1) the morphological method, 2) the anatomical method, 3) the division of seeds into diseased and sound ones according to their specific weight, 4) the biological and physiological methods, and 5) photographing of seeds by means of invisible rays.

The most promising of all those methods is the biological method. But it appears also to lack the required simplicity. A high degree of precision can be reached by using the anatomical method, but not without considerable waste of time in making the preparations and a great skill is required from the worker.

The technique proposed by Skvortzov (4) for determining the mycelium in grains consists in preliminary treatment of embryos by alkali and their subsequent staining with aniline-blue. The method compares favourably

with the anatomical method, since it essentially simplifies the analysis. Its drawback, however, is that one cannot be sure whether the mycelium available in the grain has been detected entirely in that way. It is known from the works of Klushnikova([3]), Bubentzov and others as well as from our own investigation that the spawn of the loose smut is met with not only in the embryo, but also in the fruit and seed coats and sometimes in the endosperm and other grain tissues. Erroneous conclusions are therefore apt to arise if the observation is confined to the embryo. We have made good use of that part of the technique proposed by Skvortzov which involves the alkali treatment and aniline-blue staining, and are now suggesting the method of seedling analysis. Its great advantage lies in the possibility of detecting the viable mycelium which is the source of the loose smut in the plant, since whatever tissue it may have occupied in the dormant seed its translocation with the beginning of growth is always directed toward the growth point.

Our method consists in the following:

1. Mycelium is known to penetrate into the seedling upon the germination of seed. In order properly to ascertain the ratio of viable mycelium in the seeds, the latter must be made to germinate under the conditions which are generally adjusted in the seed control laboratories where the germinating power of seeds is analyzed. Two samples each containing 100 grains are taken. After germination and when the seedlings have reached a length of nearly one centimeter they are separated from the seeds. While doing this the base of the seedling with the rest of the embryo must be grasped by the forceps, and with long seedlings it is well if their upper part is removed. The seedlings are placed in a beaker containing a 5% solution of NaOH and there they are heated.

2. Washing in water follows. The water should be changed repeatedly (5—7 times). In order that no waste of seedlings might happen on washing, the water is poured through a funnel covered with a net of thin stuff. In case of the seedlings being carried away by the water current they will meet the net and can easily be picked up with the forceps.

Washing is followed by staining in aniline-blue. 100 cm^3 water are mixed with 50 cm^3 lactic acid and 0.1 g of aniline-blue. The dyestuff solution is poured into beakers and the seedlings are kept within for 3—5 min. The amount of solution is such that the seedlings fill up its volume. For better staining heating of the solution is necessary.

3. After staining the seedlings are once washed with water. There they are cleared in lactic acid. The latter is poured into a beaker containing the washed seedlings. Discoloration is essentially furthered by heating the contents of the beaker up to the boiling point.

4. With this treatment accomplished one can immediately proceed to making the preparations. The seedlings are placed on to glass slices in a drop of the lactic acid in which they were cleared. Each of the slices receives 1—2 seedlings and the cover-glass is put upon. By slightly pressing the cover-glass one makes the seedlings to spread, the tissue becomes loose and the blue mycelium stands out on the preparation.

5. The seedlings must be placed on the slide in such a manner as not to bring their tissue in contact by pressing the cover-glass, since this would prevent the recording of infection for each of them separately. The infection ratio is determined as an average of the data obtained by analyzing two samples, each including 100 grains. From the description of this method it is obvious that it can be made a concomitant to the germination test. If the infection ratio is lower than one, more than two samples need to be taken (in order to obviate the germination of spores of external smut species

whereby their mycelium can penetrate the seeds, the disinfection of the material under test is necessary). The procedure suggested makes it possible to estimate the infection of wheat seeds by loose smut mycelium within a rather short time. After 2—3 samples, 100 grains each, have been analyzed, the infection can be established.

Simplification of the method for detecting mycelium in the tissues of a vegetating plant seems useful in that case too when breeding for plants resistant to loose as well as other species of smut is aimed at. The anatomical method used for that purpose is laborious and cannot always be applied by the breeder who has generally to deal with a great amount of material. But a proper knowledge of smut's behaviour in a vegetating plant is indispensable. Whether the cause of resistance to loose smut lies in that the contagious matter is prevented from penetrating the embryo or whether the escape of the plant from the injury might be ascribed to its being ahead of the growth of the spawn—the question cannot be answered positively by maens of field observation. We are suggesting here a more simple method for detecting the spawn in a vegetating plant. The analysis may be conducted as follows. The leaves and stems are cut into small pieces, 4—5 mm, and immersed in a 5% solution of NaOH or KOH which is then heated up to the boiling point. Thorough washing in water follows and staining with aniline-blue (the preparation of the dye has been described above). The test material is kept in the dye for 2—3 min. Then washing is repeated and the differentiation in the lactic acid is made attended by heating. The material treated is taken out from the beaker and is placed on to glass slices, 1—2 pieces on each slice. The objects are spread under the pressure exerted on the cover-glass. The tissue becomes loose and the mycelium which has taken on blue colour is distinctly visible in the intercellular spaces. The mycelium stands most forcibly out in the nodes. It is therefore the nodes which should be taken first for analysis, then the internodes and the leaves. In order to prepare the analysis without detriment to the yield, it is advisable when we have to do with valuable specimens to take lateral shoots and the leaves of the axial stem. If for some or other reason the material cannot be treated in fresh condition, it may be preserved in 80% alcohol or as a herbarium. At long last the plants can be taken from the harvested crop and allowed to wait for analysis under labels. The method which permits of detecting the mycelium in dried material is to soften the stems and leaves by boiling them in a 20% solution of NaOH for an hour. By varying the duration of alkali treatment according to the stiffness of the balm taken for analysis, we can readily detect, in the way mentioned above, the mycelium of various smut species in the tissues of dried plants.

In evolving this method we were successful to detect: 1) the mycelium of loose and hard smut in both vegetating and dried wheat plants, and 2) the mycelium of loose smut in dried plants of barley, oats, and millet.

The technique suggested can be successfully applied in seed control laboratorium for determining the ratio of loose smut infection of seeds by analyzing the seedlings. The same method can also be termed to account by those breeding stations which are concerned with growing cereal varieties resistant to different species of smut.

Voronezh Plant Protection Station.

Received
29. IV. 1939.

REFERENCES

[1] С. Т. Бубенцов, Итоги ВИЗР (1935). [2] И. П. Будрина, ibid, (1935). [3] Е. С. Клюшникова, Болезни растений, № 1—2 (1928). [4] С. С. Скворцов, Защита растений, № 15 (1937). [5] Г. П. Щеглов, Итоги ВИЗР (1936).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIV, № 2

PLANT PHYSIOLOGY

OIL CONSTANTS OF VARIOUS FORMS OF *LALLEMANTIA*

By D. J. VAKULIN and M. J. REUTMANN

(Communicated by N. I. Vavilov, Member of the Academy, 8. V. 1939)

A new plant has recently come into the scope of oil-plant cultivation in this country—*Lallemantia iberica* F. et M., fam. *Labitae*. This early ripening and drought resistant annual has soon spread over our fields and become a valuable member in the series of crops yielding quickly drying, fatty oils, a quality which only few of our crops show (*Aleurites corolata*, *A. Fordii*, *Perilla ocymoides* L.). The plant is now being studied in several points of this country; among their number, at the Botanical Garden of the State University in Odessa.

The oil of *Lallemantia* is used in preparing drying oils, paints and varnishes and its use should be attempted also for preparation of linoleum, etc. In respect of its oil quality *Lallemantia* ranks among the most valuable crops, as perilla, flax and others. Its yield of oil amounts to 37%.

The following constants have been ascribed by Richter to the *Lallemantia* oil: specific weight, 0.9356 at 20°C; freezing point,—35°C; saponification number, 185; iodine number, 162.

It can be substituted for linseed-oil in the preparation of paints. If placed upon a watch-glass for 9 days, it turns into a thick resinous film and if heated at 150° C during 3 hours it becomes dry in 24 hours.

The plant has been investigated at the Botanical garden in Odessa for several years by D. J. Vakulin (one of the authors of the present paper) who has had at his disposal a great wealth of material supplied from different points of the USSR and from abroad, and has been able to observe considerable differences between some of the forms available ([7],[8]).

1. The forms with blue flowers give small seeds of very moderate absolute weight, 2—2.5 g on the average, their number per plant being rather low.

2. The form with white flowers gives much larger seeds whose absolute weight is twice that in the blue-flowering form and the number per plant is higher.

3. The form with pink flowers approaches to the white form with regard to the above characters.

The blue form, as shown by Vakulin([7]), is very responsive to yarovization if the latter is conducted at a low temperature during a short interval (about 10 days); and the effect is still more pronounced if the yarovization lasts longer([8]).

Since the different forms of *Lallemantia* were displaying an array of essentially different characters, it seemed of interest to reveal how much

do they differ with regard to the yield of oil and its physiological constantst the more so, as no data on the oil constants for various forms of this plan; were available in the literature and the forms themselves had just been discovered.

In the literary reports, even in the most recent ones, *Lallemantia* is treated as a whole without reference to its separate forms. But the difference between the latter is so essential that now, after this has been made clear by our investigation, it is no longer a proper method to speak of this plant in general without mentioning the form concerned (blue, white, etc.). The lack of such information in the literature prompted us to proceed to the determination of oil constants for various forms of *Lallemantia*. The material for the test was selected by Vakulin. Testing was made by Reutmann at the laboratory of Technical Chemistry, Odessa State University.

5 specimens of the various forms harvested in 1938 at the Botanical Garden of Odessa were subjected to analysis.

1) No. 48—52, blue form received from Madrid in 1936.

2) AUPBI K-10, white form supplied by the All-Union Plant Breeding Institute.

3) DBS-2, white form supplied by the Don Breeding Station.

4) No. 43—60; pink form received from Penza in 1936.

5) No. 55, pink form, a strain propagated at the Botanical Garden of Odessa.

The seeds to be analyzed were ground in a hand-mill. Oil was extracted in Sexlet apparatus by means of diethyl ether. The extraction was continued until the entire oil was drawn out from the seed. The ether was expelled from the extract by heating the latter over a water bath at 100°C in a current of carbonic acid in order to prevent the oil from oxidation.

Distillation lasted until a constant weight of the oil was obtained.

After the oil thus obtained had been dried, it was turbid and separated a colloidal residue on standing. The specific weight was determined by means of a pycnometer at 15°C. The refraction was tested with a refractometer at 20°C; acid number and saponification number, according to Green. The oil readily dissolved in a neutral mixture of one volume of alcohol with two volumes of ether. The iodine number was determined according to Hueble. The dried oil was prepared from the resinate of Mn obtained by fusing colophony with manganese peroxide([2]).

The resinate analyzed afterwards was found to contain 4.8% of metallic Mn. The oil was diluted with the siccative to such an extent as to make it acquire a content of Mn equal to 1% by weight. The dilution was carried out at a temperature ranging between 200—220°C. The drying oil obtained was spread over a glass-piece. The glass was kept aslant (45°) during 40 minutes for better draining of the excessive oil and then brought to horizontal position and allowed to rest at a mean room temperature of 18° C.

Some of the samples stuck while they were examined 18 hours later, but all of them dried completely after 72 hours. The results are summarized in the table.

It may be seen from the table that here as in our previous investigations the absolute weight of seeds in the white and the pink form was found to be twice that of the blue form. The least weight of seeds was recorded in the specimens DBS-2 and AUPBI K-10. The yield of oil varied from 27.1 to 31.5%. In the previous tests ([7,8]) it amounted to 37%. The yield of fatty oil was a maximum in the pink form No. 43-60 and in the white form DBS-2.

The minimum was with the blue form 48—52 and the white form AUPBI K-10. The difference in yield between the latter was not great and so it

Samples	Form	Abs. weight of seed	Oil yield in %	Specific weight* at 15°C	Refraction	Acid number aver.	Saponification number aver.	Iodine number aver.	Condition of film after	
									18 hours	72 h.
48—52	Blue	2.4803	27.1	0.9399	1.485	3.8	195	177. 5	Condensed	Dry
AUPBI K-10	White	5.0339	28.51	0.9398	1.4845	3.75	189.8	184	Condensed	Dry
DBS-2	»	5.2580	31.5	0.9375	1.485	4.05	193.75	191.33	Not quite dry, somewhat sticky	Dry
43—60	Pink	4.9066	31.4	0.9377	1.485	4.05	195.25	196.7	Condensed	Dry
55	»	4.8128	28.02*	0.9335	1.484	2.7	192.7	197	Very sticky	Dry

was also in the experiments of 1936—1937 in which no strong departure from the average content of oil was observed among the forms tested.

Neither could any considerable deviation from the average for these three forms be noticed in the specific weight and refraction. As to the acid number, its extremes were represented by the two pink forms No. 55 and 43—60. In the latter it was 1.5 times greater than in the former. The blue form occupied an intermediate position. The white form DBS showed as high an acid value as that in the pink form No. 43—60. The saponification number showed considerable departure only in the specimen AUPBI K-10 where its value was less than in others.

The iodine number which is a most typical index of technical oils, revealed an unmistakable differentiation of the oil among the forms of *Lallemantia* tested. In the first instance we have to note that all of them had a very high iodine number ranging between 177.5 and 197. Its value was decreasing in the order: pink form specimens (196—197), white form specimens (184—191) blue form specimen (177.5). It proved thus to be a very important index for characterizing the various forms of *Lallemantia*.

Drying ability as inferred from the condition of films was not definitely related to the form. It should be noted, however, that the specimen 55 (pink form) and DBS-2 were dry after 60 hours, whereas the others were only thick at that time. After 72 hours all specimens dried.

We arrive therefore at the conclusion that the various botanical forms of *Lallemantia* differ from one another not only in their morphological biological and other characters, but in some most important physico-chemical indices as well (especially in iodine number). This serves once more to emphasize the different nature of the forms tested and their importance for special branches of industry.

Botanical Garden.
Odessa.

Received
26. II. 1939.

REFERENCES

[1] Н. Н. Любавин, Технич. химия, VI, ч. 2 (1914). [2] Benedikt, Ulzer Analyse der Fette und Wachsarten (1908). [3] Киселев, Олифа и лаки (1935). [4] А. Грюн, Анализ жиров и восков (1932). [5] Ch. Repert (1905). [6] Рушковский и Дублянская, Хим. масличн. раст. (1935). [7] D. J. Vakulin, C.R. Acad. Sci. URSS, XVIII, No. 1 (1938). [8] Д. Я. Вакулин, Тр. Одес. Гос. ун-та (1939).

* Observation made in other years showed a much higher yield of oil for the specimen 55.

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIV, № 2

AGROPHYSIOLOGY

GROWTH OF WHEAT OF VARYING FROST RESISTANCE DURING THE HARDENING PERIOD

By I. M. VASSILIEV

(Communicated by N. J. Vavilov, Member of the Academy, 16. V. 1939)

After having established that spring wheats are ahead of winter varieties if growth is made at low temperature (1), we directed our attention to varietal differences within each of both groups. These differences did not fail to put in an appearance in the previously described investigations, but there they remained in the shadow.

Now, four varieties of winter wheat (Kooperatorka, Moskovskaya 02411, *Hostianum* 0237, *Lutescens* 0329) and as many varieties of spring wheat (*Lutescens* 062, *Caesium* 0111, *Melanopus* 069, *Hordeiforme* 010) have been studied in detail. The method for comparing their frost resistance was to freeze them repeatedly in refrigerators as well as to estimate the injury and mortality among plants subjected to natural frost in winter. In accordance with the data of former investigations, the winter wheats were found to be increasingly resistant to frost in the order in which they have been enumerated above, the least resistant Kooperatorka and the most resistant *Lutescens* 0329 being the extremes of this series. Among spring wheats the variety *Hordeiforme* 010 unmistakably exceeded the others in frost resistance, while these latter, *Lutescens* 062, *Caesium* 0111 and *Melanopus* 069, showed almost equal tolerance to frost. At least the method applied failed to detect any definite differences in their behaviour.

Growth of Winter Wheats of Varying Frost Resistance Sown as Usually toward the End of Summer. Sowing was made August 26, 1938, out-of-doors in wooden boxes of 20 kg soil capacity. Each box contained all the varieties tested. The weather continued very warm until the middle of September. Mean daily temperature was above +15°C. In the time afterwards the temperature never attained this level and kept decreasing. The plants were making growth, albeit a slow one, till the middle of November.

Toward the middle of September they had already four leaves developed on the main shoot and began tillering. Their habitus gave scarcely any sign for differentiation between the varieties All of them bore equally grown leaves at that time. But by and by the difference in this particular became apparent and after 7 leaves had been developed on the main shoot they no longer showed equal size in different varieties. The wheat plants represented in Fig. 1 are as they looked on November 21. The lower leaves which had died away, are removed in all plants.

The largest leaves has Kooperatorka, then follows Moskovskaya 02411. The varieties *Hostianum* 0237 and *Lutescens* 0329 differ but little from one another with respect to the size of leaves but they are strikingly different from both Kooperatorka and Moskovskaya 02411 in bearing much smaller leaves.

Growth of Wheats of Varying Frost Resistance Sown Late in Autumn. The modification of this experiment consists in later sowing of plants, October 14. Since the temperature was

Fig. 1. From left to right: Kooperatorka, Moskovskaya 02411, *Hostianum* 0237, *Lutescens* 0329.

Fig. 2. From left to right: Kooperatorka, Moskovskaya 02411, *Hostianum* 0237, *Lutescens* 0329.

slowed down from the very start, the difference in growth intensity between the varieties came to light at earlier phases. The plants as they looked on October 21 are represented in Fig. 2. Toward that time each of them possessed 2—3 secondary shoots and bore 4 leaves on the main shoot.

The growth difference between the varieties is fairly distinct. The less frost resistant the variety, the smaller are the leaves which it bears.

Spring wheats were also involved in this experiment. Their several representatives are shown in Fig. 3 as they looked on November 13.

Conspicuously small leaves are borne by *Hordeiforme* 010 which, as it has been mentioned above, is also the most resistant of all the spring varieties tested.

Growth of Winter Wheats of Varying Frost Resistance on Their Greenhouse Cultivation in

Winter at Low Temperatures. The plants were sown into boxes on an open plot October 8. They remained there until November 25 whereupon they were transferred to a cool greenhouse with a constant temperature of about +5, +8°C. While on the garden plot the plants had no time to send forth secondary shoots; neither did the latter appear during the period in the greenhouse and the plants remained without any but the axial shoot. February 2 they looked as in Fig. 4.

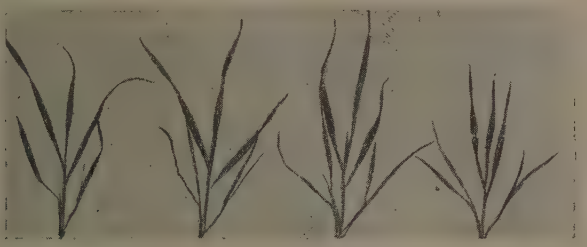


Fig. 3. From left to right: *Lutescens* 062, *Caesium* 0111, *Melanopus* 069 and *Hordeiforme* 010.

There is an obvious difference between the varieties with regard to growth intensity, which is especially conspicuous if the leaves of the third or a higher node are compared. Each of the respective leaf nodes was set simultaneously in all the wheats tested, but the growth intensity was not equal and after

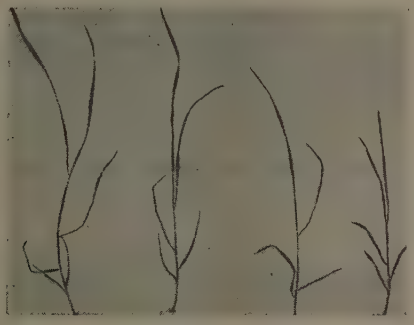


Fig. 4. From left to right: Kooperatorka, Moskovskaya 02411, *Hostianum* 0237, *Lutescens* 0329.

a certain interval had elapsed, the size of the leaves was gradually decreasing in the order in which the respective wheats are arranged in Fig. 4.

Similar picture was observed in a parallel test in which the plants sown 8. X were kept in a greenhouse at as high a temperature as about +20°C, until the phase of fourth leaf formation was reached whereupon the temperature was reduced to that in the previous experiment. While the temperature continued high, no growth differences could be noticed between the varieties. The differentiation began however with the advent of low temperature. Then the more frost resistant varieties were observed to make slow growth.

Before we finish we will refer to some measurements of the leaf length.

The data of the table show to what extent the differences in the growth response of different wheat varieties to low temperature are significant. Especial significance belongs to the data on the 4th node leaves, since the latter were forming when the temperature had reached its lowest point in the experiment.

Length of Leaf Blades in Spring and Winter Wheats Sown September 14 on a Garden Plot (Measurements made December 4 1938. Average length in cm from 15—20 measurments)

Variety	2nd leaf from below	3rd leaf from below	4th leaf from below
Spring			
Lutescens 062	10.22 ± 0.23	13.21 ± 0.37	13.88 ± 0.27
Caesium 0111	11.63 ± 0.22	12.16 ± 0.24	13.56 ± 0.38
Hordeiforme 010	10.26 ± 0.21	11.58 ± 0.36	11.57 ± 0.28
Melanopus 069	10.33 ± 0.19	12.37 ± 0.23	14.28 ± 0.30
Winter			
Kooperatorka	8.13 ± 0.21	8.41 ± 0.19	8.27 ± 0.24
Moskovskaya 02411	7.39 ± 0.23	7.60 ± 0.22	7.41 ± 0.17
Hostianum 0237	7.49 ± 0.20	7.13 ± 0.19	6.68 ± 0.13
Lutescens 0329	7.01 ± 0.18	7.0 ± 0.19	6.11 ± 0.14

Conclusions

1. Different varieties of both winter and spring wheats are not alike with regard to growth intensity at low temperature. Within each of these groups the more frost resistant varieties make slower growth in response to low temperature.

2. The fact that the frost resistant varieties respond to low temperature, which is also prevailing during the hardening process, by decreasing their growth intensity appears to be one of the most important factors in their resistance to frost.

Institute of Grain Economy
in the Regions outside the Black-Soil
Belt.

Received
17. V. 1939.

REFERENCES

[1] I. M. Vassiliev, C. R. Acad. Sci. URSS, XXIV, № 1 (1939).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIV, № 2

CYTOLOGY

DIMORPHISM OF THE CARYOTYPE IN *CHRYSANTHEMUM CORONARIUM*

By V. GLOTOV

(Communicated by A. A. Richter, Member of the Academy, 19. V. 1939)

Caryological study of the genus *Chrysanthemum* (tribe *Anthemideae*, fam. *Compositae*) has shown certain phenomena which did not fully fit into the principles of cytology. In particular, while studying samples of *Chrysanthemum coronarium* L. deviations from the «law of specificity of chromosome sets» were discovered.

This principle which was established by Boveri in 1909 and with minor modifications and additions survived until the present days, holds that the number and shape of the chromosome is constant for each species. This is the attitude assumed so far nearly by all the biologists (Belar, Hartmann, Wilson and others).

During the last years data were accumulated which were in disaccord with this concept, namely that the caryotype, i. e. number and shape of the chromosomes, their thickness, length, ratio of arms, sizes of the satellites, etc. is subject to modification nearly in the same degree as the phenotype, the manifestation of which is highly dependent on external conditions. M. Navashin, Levitsky, Avdulov, Breslavetz, Darlington (1927) produced evidence in support of this.

We cannot any more categorically support the law of «specifity of chromosome sets» as formulated earlier and should say that «chromosome shape is undoubtedly very plastic and subject to various minor changes both in different tissues of the organisms and in different cells belonging to a common tissue» (1). M. Navashin spoke in the same sence: «absolute size (length, thickness, mass) may not serve as dependable characteristic since they are strongly modified under the influence of a number of accessory factors». In the study of the genus *Chrysanthemum* several facts were found which support these theses, both with regard to chromosome shape and number.

Data on caryology of *Ch. coronarium* are presented in this paper. This species belongs to *Eu-Chrysanthemum* and grows wild in the Mediterranean region. We have studied 30 samples of *Ch. coronarium* which were obtained from different geographic localities and 3 samples of clear natural hybrids between *Ch. coronarium* and *Ch. carinatum*.

All the samples were germinated in Petri dishes, planted into pots and fixed at suitable age. The material was fixed in all cases with the chondriosomal fixative of Levitsky; it consisted of equal parts of 1% solution of

chromic acid and 40% formalin. Slides were stained with iron haematoxylin after Heidenhain.

Measurements of the chromosome length was done by means of the Levitsky's projection method; the magnification applied was 5000 diametres. Calculation of the volume of the stainable portion of the chromosome (under the application of above-fixing fluid) was done with the aid of the following formula:

$$v = \frac{4}{3}\pi R^3 + \pi R^2 (L - D)$$

where R, radius of the chromosome; D, diameter of the chromosome; L, length of the chromosome. The formula was constructed from the formula for the volume of the sphere $\left(\frac{4}{3}\pi R^3\right)$ and that for the volume of the cylinder (the remaining cylindrical part of the chromosome)*.

For computation of the thickness of the chromosomes were taken weighed average values from two measurements.

From the very beginning of the work on *Chrysanthemum* I have adopted geographical criterion for the study of caryotypes. The idiogram of the species was deduced not from a study of a single sample taken by chance but from as many samples as possible and, besides, obtained from different localities of the basic area of the species in question.

The caryotype of a species does not present a uniform aspect. Uniformity of the caryotype may be admitted in young species only or strictly limited though relict endemics. Species which are widely distributed, getting into most varied climatic and ecological conditions necessarily react to them producing new caryoraces through various changes of the caryotype (Hagerup, Tischler and others) or mutating by means of genovariation. To establish the real caryotype of any species it is necessary to study it in all its polymorphism; otherwise erroneous conclusions may be reached. Investigation of *Ch. coronarium* have corroborated this thesis.

Out of the 30 samples of *Ch. coronarium* investigated by me 25 had a caryotype which was normal and typical of this species ($2n=18$); the chromosomes were typical, large and comparatively thin. The remaining 5 samples had, aside from the normal caryotype, a somewhat modified one.

These changes were expressed in the following: in the one and the same rootlet, often in the one and the same section, both modified and normal caryotypes are met with. The difference between them was sharp enough to distinguish between the two caryotypes even when slides were superficially inspected.

The 1st type is the normal caryotype of the species in question, $2n=18$, chromosomes are long and thin, which is typical with a few exceptions of the whole genus *Chrysanthemum*. All the chromosomes are two-armed with well expressed constrictions and with rather large achromatic interruptions.

Type 2 has the same chromosome number, $2n=18$, but the chromosomes are greatly shortened and thickened (Fig. 1 and idiogram 1). Constriction are less expressed and no achromatic interruptions are present.

Drawings were made with Abbe camera lucida (of C. Zeiss) on the level of the working table; immersion lens 120 x and compensating eye piece 15 x were used, the actual magnification being equal to 3460 diameters. Occasionally plates are met with which contain chromosomes of intermediate form both with respect to thickness and to length. This type was not as clear-cut as the 1st and 2nd types.

---

* The formula has been constructed by Eng. S. F. Jigalov to whom my thanks are due.

Occurrence of the two types was correlated neither with their localization in definite tissues of the root nor with the age of the latter. The two types were found in various layers of the periblem and in the dermatogene. The type 1 is not found in the plerome whereas the type 2 sometimes occurs. It should be noted that the two types occur approximately in equal numbers. The two types are met with both in younger parts of the rootlet, near the rootcap, and in its older parts. Occurrence of the one or of the other type is not connected with the size of the cells as noted by Levitsky and Belar who reported the chromosomes to be longer in larger cells and vice versa («their size may vary although not very much»). The cell size is approximately the same in the two cases; however, no special measurements were made. The noted changes are of another kind than those which are familiar to any cytologist i. e. shape differences in plerom, periblem and dermatogene.

They have nothing in common with the phenomenon described by M. Navashin, namely, that chromosomes are somewhat longer in rootlets of adult plants than in those of young seedlings. Such changes are not as strong as those reported here and, besides, there is some ground for them.

To attain more objectiveness and exactitude I have made measurements of the chromosome length and chromatic volume in 2 most characteristic plates of the 1st and 2nd types in which the chromosomes were arranged most favourably (Table 1).

Table 1

Comparison between the Chromosome Length and Volume in the 1st and 2nd Types

Chromosome pairs No.	Length of the chromosomes in μ						Volume of the chromosomes in μ³					
	Arm *a*		Arm *b*		Total length		Arm *a*		Arm *b*		Total volume	
	Type 1st	Type 2nd	Type 1st	Type 2nd	Type 1st	Type 2nd	Type 2nd	Type 1st	Type 2nd	Type 1st	Type 2nd	Type 1st
1	3.24	2.16	3.84	2.31	7.08	4.47	0.175	0.485	0.195	0.52	0.37	1.005
2	3.76	2.405	4.27	2.44	8.03	4.845	0.2	0.54	0.23	0.55	0.43	1.09
3	3.675	2.265	4.595	2.535	8.27	4.8	0.19	0.51	0.27	0.58	0.46	1.09
4	4.18	2.385	4.59	2.46	8.77	4.845	0.225	0.54	0.245	0.56	0.47	1.1
5	4.06	2.57	5.13	2.78	9.19	5.35	0.215	0.575	0.28	0.64	0.495	1.215
6	4.665	2.245	4.8	2.845	9.465	5.09	0.25	0.505	0.26	0.655	0.505	1.16
7	3.245	2.425	6.065	2.945	9.31	5.37	0.17	0.55	0.33	0.68	0.5	1.23
8	4.09	2.245	5.995	3.425	10.085	5.37	0.22	0.51	0.325	0.795	0.545	1.305
9	4.7	2.75	6.545	3.475	11.245	6.225	0.25	0.63	0.355	0.81	0.605	1.44
	Total:				162.89	93.33					8,76	21.27

These data support the reality of the 2 types of plates in *Ch. coronarium*. Mathematical analysis—calculation of v, σ and m_d fully corroborates this conclusion. Thus, the mean value for the chromosome length (m_d) is 18.2, that for the volume—27.1. As may be seen, the values m_d are in all cases far beyond the limit so that the difference in size and volume of the chromosomes is fully significant.

The chromosomes of the 1st and 2nd type may be arranged according to length and ratio of arms in the following way (Table 2).

Table 2

Groups	Chromosome number	
	Type 1	Type 2
1. Small isobrachial chromosomes	—	2
2. » chromosomes with the ratio of arms=1 : 1.2	2	—
3. Medium-sized isobrachial chromosomes	2	4
4. » » chromosomes with the ratio of arms=1:1.1—1:1.3	4	8
5. Large isobrachial chromosomes	2	—
6. » chromosomes with the ratio of arms = 1 : 1.2	2	—
7. » » » » » » » =1 : 1.4	4	4
8. » » » » » » » =1 : 1.8	2	—
Total	18	18

It should be noted that distribution of the chromosomes according to groups is a rather arbitrary one, as no sharp distinction between large, medium and small chromosomes exists.

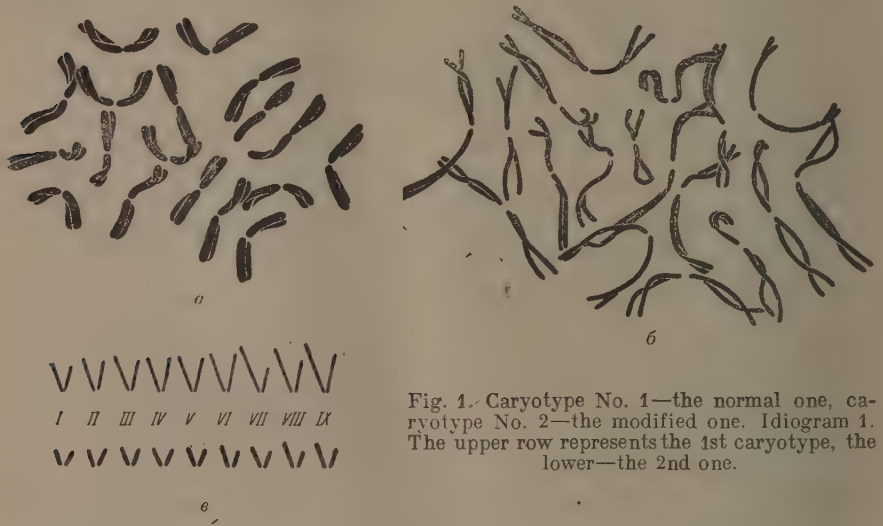


Fig. 1. Caryotype No. 1—the normal one, caryotype No. 2—the modified one. Idiogram 1. The upper row represents the 1st caryotype, the lower—the 2nd one.

From Table 2 it is evident that the distribution of the chromosomes in groups is somewhat changed in type 2. Thus, the large chromosomes with the ratio of arms=1 : 1.4 retained their position in both cases causing only a change in the total length of the chromosomes, while the remaining large chromosomes in the 1st type fit into the medium groups of heterobrachial chromosomes in the 2nd type. Finally, the number of isobrachial chromosomes of medium and small size became strikingly increased. Thus, along with general reduction of the chromosome length, a stronger contraction of the longer arms as compared with the shorter ones is observed and, consequently, a decrease of these ratios. A similar disproportional contraction of the arms was observed after subjecting rootlets to cooling after treatment with chloral hydrate.

Expressed in per cent, the volumes are as followed: reduction of the longer arm equals on the average 45.1%, that of the shorter one 38.8% only. The volume of the chromosomes in the 1st type varies from 0.37 to 0.605 μ^3, whereas in the type 2—from 1.005 to 1.44 μ^3.

A rather curious aspect is gained from a comparison of the total length and the volume of all the chromosomes. In the type 1 the total length of all the chromosomes equals 162.89 μ, the volume—8.76 μ^3. In the type 2 the respective values are 93.33 μ and 21.27 μ^3.

It means that despite the significant reduction of the chromosome length in the type 2 (1.75 times as compared with the type 1) the amount of the stainable portion of the chromatin is sharply increased, 2.45 times. Such a significant increase in chromatin which undoubtedly exceeds the common intra-individual and intra-cellular variation can be hardly explained at present, unless special investigation is made.

Discussion of the Data Obtained. The principal point derived from the material presented above is the existence in *Ch. coronarium* of dimorphic caryoforms differing in size, shape and volume of the chromosomes and occurring not only within one and the same plant, but also within one and the same rootlet, in one and the same section and tissue of a rootlet.

Out of the 30 samples studied only in 5 such a dimorphism was found. In the remaining not a single case was observed. In the study of a larger number of samples of the genus *Chrysanthemum* or of other representatives of the tribe *Anthemideae* such cases of *Chrysanthemums* were observed neither by me nor by other students (Shimotomai, Tahara and others).

Such changes in chromosomes are similar to those obtained from freezing germinating seeds and young rootlets (in experiments by Delaunay) or in treating young seedlings with chloral hydrate as showm by Kagawa in wheat*. Natural shortening and thickening of the cromosomes was observed by N. P. Avdulov in *Penicillaria spicata* Willd. in a single cell near the tip of the rootlet. The origin of such a plate was due, to cite his own words, to «unknown cause». The same writer has observed in a triploid sector in Indian corn ($3n=29$) with larger cells a strong contraction and thickening of the chromosomes as compared to normal diploid cells. «Increase of the cell size evidently is not proportional to the increase in chromosome number, and plates with augmented number of the latter are arranged more closely», owing to which they become shorter and thicker. The question is thus reduced to insufficient nutrition of the cell. L. P. Breslavetz has observed in some samples of *Melandrium album* plates with thinner than usual chromosomes. The length remained, however, unchanged.

The above instances, though not fully similar to the case of *Ch. coronarium*, suggest certain idea of the change. To explain precisely the occurrence of such a dimorphism, a special additional investigation of *Ch. coronarium* is necessary. At present one may only suppose that the cause is in some disturbances or peculiarities of the physical state of the cell protoplasm. Besides, this should take place not in all cells of the rootlet, but in individual cells distributed by change and not in every sample of the species, although all the samples and individual plants were grown under similar conditions and cytological material was treated simillarly. Levitsky says that «modificatory variation of the chromosomes is caused by the influence of external conditions and gradually annihilated upon cessation of their action. Shortening and thickening of the chromosomes takes mostly place; such

* A similar contraction and thickening of the chromosomes I have found in *Chrysanthemum* treated with colchicine.

changes of the chromosomes sometimes occur in isolated cell, mostly peripherically located, and, evidently, under wholly normal conditions».

It may be thought that chromosome shape is in part determined by the physical conditions of the cytoplasm at the moment of cell division, mainly, by its viscosity. When viscosity is less, the chromosomes should be longer and thinner than when density of the cytoplasm is greater. Viscosity of the protoplasm is very variable and depends on a number of conditions. Thus, it changes even during the cell division, being greater toward prophase and during the whole process of mitosis to drop after it is over. In the center of the cell viscosity is greater than on the periphery. Viscosity increases with decrease in water content ([5]).

It is evident that in some way still unknown to us «the surrounding medium changes chemistry of the lining cell, its colloidal and other properties which are reflected on the structure of the plasm, of the nucleus of the chromosome and its individual parts, these changes occurring through the cytoplasm» ([6]). In the case of *Ch. coronarium* the chromosome dimorphism is evidently due to a change in physical properties of the contents of separate cells, possibly connected with hybrid origin of these forms which are, therefore, not fully stabilized.

The second peculiarity observed in this case is that, in connexion with changes in the karyotype toward the shortening or thickening of the chromosomes, took place a striking increase in the chromatin mass—from 8.76 to 21.27 μ^3. No explanation of this phenomenon without special study is possible. It may be just suggested that the chromatin mass also is modified in each cell and rather strongly, too. In order to clear up the cause of this phenomenon we have undertaken further studies on micro- and macrosporogenesis, hybridization between modified and normal forms and on repeated caryological study of the same samples.

Received
29. IV. 1939.

REFERENCES

[1] Н. И. Авдулов, Сб. Тр. Сар. сел. оп. станции (1937). [2] Беляр (1934). [3] Бляхер (1935). [4] Л. П. Бреславец, Введ. в цитологию (1932); Planta, **7** (1929). [5] Э. Вильсон (1936). [6] Гришко и Делоне (1938). [7] Г. А. Левитский, Тр. по прикл. бот., ген. и сел., XXVII, вып. 1 (1931). [8] С. Г. Навашин (1920 и др.). [9] М. С. Навашин, Усп. совр. биол., III, вып. 1 и др. (1934). [10] Натали, Магржиковская и Хвостова (1935). [11] H. Bleier, ZS. f. Zellforsch., 11 (1930). [12] Darlington, Rec. Adv. of Cyt. [13] F. Kagawa, J. of Col. Agr. Imp. Univ. of Tokyo (1929). [14] H. Rohweder, Bot. Zbl. A, Beih. **54** (1936). [15] K. Sakai, Jap. J. Gen., **9** (1934). [16] N. J. Shimotomai, Sci. Hiroshima Univ., 2 (1933). [17] N. J. Shimotomai a. T. Takemoto, Bot. Mag. Tokyo, **50** (1936). [18] N. J. Shimotomai a. Hara, Pflanzen und Tiere, **3** (1935). [19] N. J. Shimotomai, J. of Sci. of the Hir. Univ., **1** (1932). [20] M. Tahara, Bot. Mag. Tokyo, **29** (1915). [21] M. Tahara, J. Coll. Sci. Imp. Univ. Tokyo, **43** (1921). [22] M. Tahara a. N. Shimotomai, Sci. Rep. Tohok. Imp. Univ., Ser. 4, **2** (1927).

Comptes Rendus (Doklady) de l'Académie des Sciences de l'URSS
1939. Volume XXIV, № 2

EXPERIMENTAL MORPHOLOGY

ON THE CHARACTER OF CELL CHANGES IN NEURAL PLATE INDUCTION

By G. V. LOPASHOV

(Communicated by I. I. Schmalhausen, Member of the Academy, 19. V. 1939)

Resuming my experiments on the specificity of the action of inducing substances ([8], [9]) I obtained a number of data which, having at first glance no direct relation to the purpose of the experiments, permit, however, of a clearer understanding of the problem of induction specificity and of dividing the process of induction into its separate components. During the summers 1936 and 1937, I carried out more than two thousand experiments on the induction in the ectoderm of gastrulas of *Triton taeniatus* by various dead tissues taken from amphibian embryos; the experiments were started chiefly on entire embryos. However, typically formed rudiments were induced only in separate cases while in the majority of cases there occurred only insignificant changes in the ectoderm above the inductor, or else, the cells changes, but f a i l e d t o c o m b i n e i n t o f o r m e d o r g a n s. As the embryos were fixed at the stages of tail bud, beginning of pigmentation didactylous limbs, or, in separate cases, still later, it proved possible to follow the development of such «underdeveloped» rudiments and to ascertain their nature.

The pictures observed at the first two stages are in many respects similar. The rudiments induced can be divided into the following groups, corresponding in general features to the classification of inductions by dead tissues given by Holtfreter([4]): 1) formed fragments of brain (inclusive of those with eyes); 2) brain with cells irregularly disposed, lacking typical structure; 3) groups of separate elongated cells resembling brain cells, especially at an earlier stage; 4) a layer of cylindrical cells lying below the exterior epithelium and sometimes, at a later stage, passing into small vesicles, or placodes. Different types of induced rudiments may also appear together, following definite rules: 1) a well differentiated brain is not accompanied by rudiments of other types; 2) scattered groups of brain cells are usually formed together with cylindrical epithelium; 3) different forms of the latter are often formed singly. Besides that, at earlier stage there is observed another type of cell changes which seems to be essentially different from those described above. In the region of induction the epidermis is thickened and becomes polystichoeous; later on this thickening usually disappears. It may be ascribed to the action of the dead tissues on the proliferation of epidermal cells, similar to that exerted by anlages of live sense organs on the ectoderm([9]), the latter showing no qualitative changes. Where the epithelium is thickened,

rudiments of limbs are sometimes formed; the epithelium of these limbs originates, as it seems, out of the thickened epithelium.

At later stages when the cells become more differentiated, the histological significance of the various groups of induction becomes clear. The brain rudiments, the typical, as well as the loose ones, differentiate into neural cells which in the loose brain keep apart. As to the cylindrical epithelium and the separated cells which at earlier stages have a similar appearance, they are transformed into a number of cell types which in normal development are the derivates of the neural plate and especially of the neural folds (from the latter originate cells of ganglions, pigment cells, mesenchyma, cartilage, cells of interior meninges and some other types of cells) ([2],[11—14]). Sometimes, near the epidermis or in connexion with it, lie separate cells resembling brain cells; in some cases, small ganglions are developed; from the points of induction flows mesenchyma, and now and then, opposite these points, lie small bits of cartilage (it always lies on the intestine); a great number of pigment cells are always formed; moreover, in some cases, vesicles and groups of cells are formed whose character defies accurate determination; I find it possible to assume, however, that they consist of other derivates of the neural plate and of its folds, such as cells of the brain roof of meninges, etc. The ganglions, pigment cells, mesenchyma are also formed with a well differentiated brain; however, the course of their development in such cases is different, since they seem to be separated directly from the brain roof([11]), as in the case of normal development.

The facts described above mainly agree with those of Holtfreter([4]) and confirm his data. It is essential to note that however may be built the induced rudiments, the cells of which they consist are always related to the same types which are formed at those stages in normal development. Separate cases differ from one another in that only certain types of cells arise from the given series of various cells. The origination of cells of new types alone is still unsufficient for the normal formation of the arising rudiments. «Histological» differentiation is, in many cases, realized without «organological» differentiation([4]). In the majority of cases, a normal j u n c t i o n of the cells is absent here; the neural cells keep apart without combining into a formed brain. The remaining products of the neural plate and folds which arise in the absence of brain, or in the presence of a loose brain, fail in such cases to find normal place, without connexion with the brain lie ganglions, and are formed isolated vesicles. In these cases the heterogeneous cells seem to be formed not in the usual spatial relations; the movement of the cells is greatly hampered—this is seen even from the fact that in a number of cases, the neural cells both in groups and singly keep sticking to the epidermis.

Now, what is responsible for the formation of various types of rudiments induced by dead tissues ? As early as in his first work on the induction by dead tissues, Holtfreter([3]) ascertained that induction requires a definite amount of time. The majority of my experiments was started at a temperature of 20°C and higher. The dependence of the rate of development of embryos on temperature is very great: thus, in *Triton alpestris*([7]) gastrulation at a $t°=$ $=9—11°C$ takes up to 70 hours, while at a $t°=20°C$ it takes only 20 hours. Only by the end of the experiments described did I pay attention to the fact that the result of induction may be connected with its d u r a t i o n, and I began to run experiments at low temperature; as a result both number and quality of inductions sharply increased. Considering it necessary to clear in more detail the question about the character of the action of temperature, I organized during the autumn 1938 a small additional series of experiments at low temperatures. The objects studied were *Pleurodeles waltli*, *Triton vittatus* and *Amblystoma mexicanum*. At a temperature of

8—11°C, the early gastrulas of *Triton vittatus* gave 100% of induction under the action of killed tissues. In separate cases, by greatly retarding development I succeeded to obtain induction of the brain, both by live and by dead tissues at the stage of a late gastrula when under usual conditions I was no more able to obtain them. But later, the retarding produced no effect.* Only at the stage of gastrula the ectoderm is capable of transforming into brain cells. If the ectoderm is maintained at this stage for a sufficiently long time, it is possible to call forth in it an induction of the brain. But after that, and probably by abrupt changes, the cells pass to the following phase of their development—the stage of neurula when they cease to be able to form any neural cells. Lastly, I wish to point out that the sensitivity of different species to inductional influences of dead tissues greatly differs.

The results of the experiments described are in accordance with those of Holtfreter's work([5]) who has shown that ectoderm isolated in vitro at the stage of gastrula loses its aptitude to develop into brain cells at the age of a final gastrula, and into neural folds derivates at the age of a neural. They agree also with the results of my experiments (unpublished) which have shown that also in an entire embryo of the newt the loss of the abilities described takes place at the same terms. Experiments of induction at a low temperature allow to make a further conclusion: to produce changes in the cell a certain d u r a t i o n of the inductor action is necessary which should be applied before the cells have reached the following stage of their development, at which such an induction ceases to be possible. This allows the assumption that the change of cells is connected with the accumulation in them of a certain amount of inducing substances, the function of these substances being to change the character of the cells. If the cells have had time enough to obtain a sufficient quantity of these substances by the moment of their age change, they transform into cells of those types which originate at the time of this age change; whereas those cells which have received a smaller amount of these substances, develop into cells of types which originate at later stages. It is just this way that the cells of the brain and neural folds are related to each other. The results of the process of induction may vary qualitatively or quantitatively depending on the age of the cells being induced, as well as on the intensity of the action of the inductor and the duration of its action; the latter in its turn depends on the temperature at which the development proceeds**. The principles described are of a particular character, because the origination of the cells of some or other types is connected in other cases with the specificity of the inducing substances ([1], [6], [8]).

The process of formation of rudiments as a result of induction is not exhausted by the formation of cells of different types. It is necessary that the transformed cells be joined to form a continuous layer, since without this linkage they are not combined into a formed organ, there is no influence of homogeneous cells on each other without which their structure, their differentiation is not entirely typical and a number of further influences of some cells upon others resulting in their qualitative transformations cannot be accomplished***. It is not clear why the linkage of the cells during delayed

---

* It is a matter for regret that the temperature experiments described were run with imperfect methods. The operated embryos were placed on a can containing ice. It is desirable to repeat these experiments using a thermostat.

** Comp. the rôle of temperature during lens induction ([10]).

*** The necessity of the cell contact for the normal formation of the organ was emphasized by Poležajew ([15]). He considers, however, the said contact as a whole and does not discriminate its functions for the formation and distribution of various cells, in particular during the processes of regulation and the differentiation of various newly formed cells. I believe that the processes which take place in both cases differ substantially. Consequently the significance of the contact should be quite different in either case.

induction is lost; inasmuch as such pictures were usually observed chiefly in retarded induction by dead tissues, I find it possible to assume that the absence of a close and regular contact with the induced tissue, characteristic of live inductors, results in one cell being changed, while another one lying nearby remains unchanged or else changes in another way; this results in the formation of groups of separate cells.

Institute of Experimental Biology.
Academy of Sciences of the USSR.
Moscow.

Received
25.V.1939.

REFERENCES

[1] C. D. van Cleave, Physiol. Zool., **11**, 168—177 (1938). [2] G. P. Du Shane, J. exp. Zool., **72**, 1—31 (1935). [3] J. Holtfreter, Arch. Entw. Mech., **128**, 584—633 (1933). [4] J. Holtfreter, ibid., **132**, 225—306 (1934). [5] J. Holtfreter, ibid., **138**, 163—196 (1938). [6] Ch. Hsiao-Hui, Biol. Zbl., **58**, 472—480 (1938). [7] F. C. E. Knight, Arch. Entw. Mech., **137**, 461—473 (1938). [8] G. Lopashov, Biol. Journ., **5**, 463—488 (1936). [9] G. Lopashov, C. R. Acad. Sci. URSS, XV, 286—288 (1937). [10] F. E. Lehmann, Arch. Entw. Mech., **134**, 166—199 (1936). [11] C. P. Raven, ibid., **125**, 210—292 (1931). [12] C. P. Raven, ibid., **129**, 179—198 (1933). [13] C. P. Raven, ibid., **134**, 122—146 (1936). [14] L. S. Stone, J. exp. Zool., **35**, 421—496 (1922). [15] L. W. Poležaew, Biol. Journ., **5**, 489—502 (1936).

33

PRIX D'ABONNEMENT

Un an—36 livraisons . . . $ 15.00. Frs. fr. 250.00. £ 2.8.0
Six mois—18 livraisons . . $ 7.50. Frs. fr. 125.00. £ 1.4.0
(Séparément $ 0.50. Frs. fr. 8.00. £ 0.1.6 le numéro)

URSS—MEŻDUNARODNAJA KNIGA, 18, Kuznetskij Most, Moscou.

U. S. A., NORTH and SOUTH AMERICA—BOOKNIGA INCORPORATED. 255 Fifth Avenue. New York, N. Y.

ENGLAND —W. H. SMITH & son, Ltd. Strand House, Portugal Str. London W. C. 2.
—COLLET'S Foreign Department, 67 Great Russel Street, London W. C. I.

FRANCE —MESSAGERIES HACHETTE. Service Abonnements. 111, Rue Réaumur. Paris, 2e.

SUISSE —Buchhandlung STAUFFACHER (Genossenschaft). Lutherstrasse, 6. Zürich.

BELGIQUE —OBLA (Office Belge Littéraire et Artistique). 6/8/10, rue Duquesnoy. Bruxelles.

LIETUVA —«MOKSLAS». Knygynas. Laisves Aleja, 46. Kaunas.
—SPAUDOS FONDAS. Laisves Al. 62. Kaunas.

TURQUIE —LIBRAIRIE HACHETTE. Succursale de Turquie. 469, Avenue de l'Indépendance. Boite Postale Pera 2219. Istanbul.

DEUTSCHLAND —KOEHLER & VOLCKMAR, A/G. Russland-Abteilung. Postfach 173. Leipzig C. I.

POLSKA —GEBETHNER & WOLFF. Filja I. Krakowskie Przedm. 15. Warszawa.
—ROSSICA. Chmielna 5. Warszawa.

PALESTINE —SAFRAN CO. P. O. Box 223. Binyan Aziz Khayat, Haifa.

LATVIJA —VALTERS un RAPA Akc. Sab. Teatra iela 11. Riga (Pasta tek. rek. № 675).
—K/s «LAIKMETS». Terbatas iela 37. Riga.

SVERIGE —C. E. FRITZE's Kgl. Hofbuchhandlung. Fredsgatan 2, Stockholm.
—A. B. WENNERGRENS WILLIAMS Box 657, Stockholm I.

AUSTRALIA —MODERN PUBLISHERS Pty, Ltd., 312 Rawson Chambers, Pitt Street. Sydney.

BULGARIA —T. F. TSCHIPEFF. Buchhandlung Boulevard Dondoukoff 20/22, Sofia (Postscheckkonto-Sofia 540).

NORGE —JOHAN GRUNDT TANUM. Tidligere Aschehougs Boghandel. Karl Johans gt. 43. Oslo.

ROMANIA —Biroul Central de Presa «LUVRU». Strada Danielopol, 1, Bucuresti I.

EESTI —TÖÖKÖÖL, Pärnu mnt. 28, Tallinn, Reklaamkeskus Reka, V. Posti I. Tallin.

DÄNEMARK —S. U. V. s Boghandel. Frederiksbarggade 42¹. Kopenhagen. K.

NEDERLAND—Bookhandel Nieuwe Cultuur en uitgeveriy, Valeriusstraat, 76, bv. Amsterdam.

Les «Comptes Rendus», largement expédiés á l'étranger dans les centres scientifiques les plus importants et dans les bureaux de nombreuses revues, sont le seul organe scientifique en URSS, donnant tous les dix jours un compte rendu des plus récentes acquisitions de la science soviétique.

Prix $ 0.50